The potential energy of a collection of N charged particles

Suppose the N particles are located at the terminus of the vectors $\vec{r}_i$ and carry a charge q_i. The potential at a point $\vec{r}$ generated by particle 1 is

$$\phi(\vec{r}) = \frac{q_1}{\left|\vec{r} - \vec{r}_1\right|}$$

and the potential energy of the remaining N-1 particles interacting with particle 1 is

$$V_1 = \sum_{i=2}^{N} \frac{q_1 q_i}{\left|\vec{r}_i - \vec{r}_1\right|}$$

Particle 2 generates the potential

$$\phi(\vec{r}) = \frac{q_2}{\left|\vec{r} - \vec{r}_2\right|}$$

and the potential energy of the remaining N-2 particles interacting with particle 2 is

$$V_2 = \sum_{i=3}^{N} \frac{q_2 q_i}{\left|\vec{r}_i - \vec{r}_2\right|}$$

Note that particle 1 is excluded because its interaction with particle 2 has been included in V_1. In a similar way V_3 is

$$V_3 = \sum_{i=4}^{N} \frac{q_2 q_i}{\left|\vec{r}_i - \vec{r}_3\right|}$$

Continuing we find the total potential energy

$$V = \sum_{i=1}^{N-1} V_i = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{q_i q_j}{\left|\vec{r}_i - \vec{r}_j\right|}$$

This is the familiar sum over all pairs of particles and is often written as

$$V = \sum_{i>j}^{N} \frac{q_i q_j}{\left|\vec{r}_i - \vec{r}_j\right|}$$

where *i* does not equal *j*.

Exercise

Calculate the potential energy of a system of four particles with the following charges and locations. The charge e is the proton charge and the length a is 2 Angstroms. Do so in the SI, and atomic unit systems.

particle	*charge*	*x*	*y*	*z*
1	e	a	0	0
2	e	$-a$	0	0
3	$-e$	0	a	0
4	$-e$	0	a	0