Orbital angular momentum operators

The angular momentum of a particle located at the terminus of a vector $\vec{r}$ and moving with a linear momentum $\vec{p}$ is given by $\vec{L} = \vec{r} \times \vec{p}$. The time rate of change of this vector (the torque) is given by

$$\frac{d\vec{L}}{dt} = \dot{\vec{r}} \times \vec{p} + \vec{r} \times \dot{\vec{p}} = \vec{r} \times \dot{\vec{p}} = \vec{r} \times \vec{F} = 0$$

Where $\dot{\vec{r}} \times \vec{p} = 0$ since the two vectors are parallel and the time derivative of the linear momentum is equal to the force acting on the particle, $\dot{\vec{p}} = \vec{F}$. In a central field $\vec{F}\ is\ paralell\ to\ \vec{r}$ and the angular momentum is constant.

The classical orbital angular momentum $\vec{L} = \vec{r} \times \vec{p}$ has three components

$$L_x = yp_z - zp_y$$
$$L_y = zp_x - xp_z$$
$$L_z = xp_y - yp_x$$

In quantum mechanics we replace the linear momentum with its quantum mechanical operator

$$\hat{\vec{p}} = \hat{p}_x\hat{x} + \hat{p}_y\hat{y} + \hat{p}_z\hat{z} = \frac{\hbar}{i}\nabla = \frac{\hbar}{i}\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)$$ where $\hat{x}, \hat{y}\ \&\ \hat{z}$ are unit vectors,

and write the operator for the orbital angular momentum in terms of its three components as

$$\vec{L} = \vec{r} \times \hat{\vec{p}} = \hat{L}_x\hat{x} + \hat{L}_y\hat{y} + \hat{L}_z\hat{z}$$

Where

$$\hat{L}_x = y\hat{p}_z - z\hat{p}_y = \frac{\hbar}{i}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$
$$\hat{L}_y = z\hat{p}_x - x\hat{p}_z = \frac{\hbar}{i}\left(z\frac{\partial}{\partial y} - y\frac{\partial}{\partial z}\right)$$
$$\hat{L}_z = x\hat{p}_y - y\hat{p}_x = \frac{\hbar}{i}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$