Introduction

One of the most important properties of a quantum mechanical system is its angular momentum. One might have anticipated this since Planck's constant *h*, the harbinger of quantum effects has the units of angular momentum. We will learn that angular momentum can be due to a particles motion relative to a point in space, the orbital angular momentum (an electron relative to a nucleus for example) or to an intrinsic property of a particle like its spin. Whatever the cause angular momenta share some very basic properties that we shall now develop.

We will first consider the classical definition of angular momentum and show that it is constant for a system with a central potential. After deducing the form of the angular momentum operator $\hat{\vec{L}}$ we show that this is also true in quantum mechanics by showing that $\hat{\vec{L}}$ and the Hamiltonian, $\hat{H}$ for a central field problem commute. We then show that the square of the angular momentum operator ($\hat{\vec{L}}^2$) also commutes with $\hat{H}$ and after considering the various commutation relationships, we can insist that the eigenfunctions of $\hat{H}$ are also eigenfunctions $\hat{\vec{L}}^2$ and one of its components, which we select to be $\hat{L}_z$ We will then discuss the eigenfunctions and corresponding eigenvalues that characterize $\hat{\vec{L}}^2$ and $\hat{L}_z$. We will see that orbital angular momentum is characterized by two integers, one for $\hat{\vec{L}}^2$ and one for $\hat{L}_z$ and that the eigenfunctions are the well known spherical harmonics, $Y_\ell^m(\theta,\phi)$ where $\ell = 0,1,2,\cdots$ and m is an integer , $-\ell \le m \le \ell$. Additionally by considering the angular momentum eigenfunction problem in a more abstract algebraic sense we discover that in addition to the integer values ℓ & m some angular momenta can be characterized by half integers allowing us to introduce the idea of the angular momentum associated with the spin of the electron. We shall then consider how we can express the angular momenta for a composite system in terms of the angular momenta of the constituent particles. Quite an ambitious agenda so let's begin.