

Average values of r^n

In subsequent discussions we will frequently need average values of r^n where $n = -4, -3, -2, -1, 1, \& 2$ and these are gathered below.

$$\left\langle r \right\rangle_{n\ell m} = \frac{n^2 a_\mu}{Z} \left(1 + \frac{1}{2} \left(1 - \frac{\ell(\ell+1)}{n^2} \right) \right)$$

$$\left\langle r^2 \right\rangle_{n\ell m} = \frac{n^4 a_\mu^2}{Z^2} \left(1 + \frac{3}{2} \left(1 - \frac{(\ell(\ell+1) - 1/3)}{n^2} \right) \right)$$

$$\left\langle \frac{1}{r} \right\rangle_{n\ell m} = \frac{Z}{a_\mu n^2}$$

$$\left\langle \frac{1}{r^2} \right\rangle_{n\ell m} = \frac{Z^2}{a_\mu^2 n^3 (\ell + 1/2)}$$

$$\left\langle \frac{1}{r^3} \right\rangle_{n\ell m} = \frac{Z^3}{a_\mu^3 n^3 \ell (\ell + 1/2) (\ell + 1)}$$

$$\left\langle \frac{1}{r^4} \right\rangle_{n\ell m} = \frac{\frac{3}{2} Z^4 \left(1 - \frac{\ell(\ell+1)}{3n^2} \right)}{a_\mu^4 n^3 \ell (\ell + 3/2) (\ell + 1/2) (\ell - 1/2)}$$