

Trigonometric Integrals

$$\int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) dx = \int_0^a \cos\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi x}{a}\right) dx = \frac{a}{2} \delta_{nm}$$

$$\int_0^\pi \cos^n \theta \sin \theta d\theta = \begin{cases} 0 & \text{if } n \text{ is odd;} \\ \frac{2}{n+1} & \text{if } n \text{ is even} \end{cases}$$

$$\int_0^\pi \cos^n \theta \sin^3 \theta d\theta = \begin{cases} 0 & \text{if } n \text{ is odd;} \\ \frac{4}{(n+1)(n+3)} & \text{if } n \text{ is even} \end{cases}$$

$$\int_0^a \cos\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) dx = 0$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \sin^3 ax dx = -\frac{1}{3a} \cos ax \left[\sin^2 ax + 2 \right]$$

$$\int x \sin ax dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax$$

$$\int x \sin^2 ax dx = \frac{x^2}{4} - \frac{x \sin 2ax}{4a} - \frac{\cos 2ax}{8a^2}$$

$$\int x^2 \sin^2 ax dx = \frac{x^3}{6} - \left(\frac{x^2}{4a} - \frac{1}{8a^3} \right) \sin 2ax - \frac{x \cos 2ax}{4a^2}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int \cos^3 ax dx = \frac{1}{a} \sin ax - \frac{1}{3a} \sin^3 ax$$

$$\int \sin ax \cos ax dx = \frac{1}{2a} \sin^2 ax$$

$$\int \sin^p ax \cos ax dx = \frac{1}{a(p+1)} \sin^{p+1} ax \quad (p \neq -1)$$

$$\int \sin ax \cos^p ax dx = -\frac{1}{a(p+1)} \cos^{p+1} ax, \quad (p \neq -1)$$