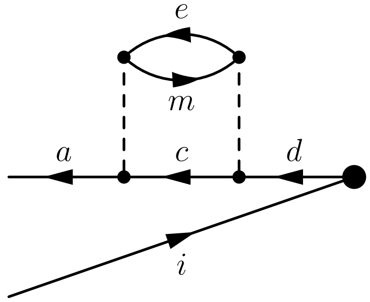
CEM 993 **Spring 2023**

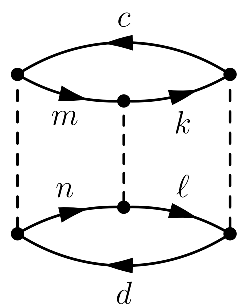
Assignment No. 4 (20 pts.) **Due Date: March 29, 2023**

1. (3 pts.) Assign weights (w_R), signs (s_R), scalar factors ($d^{(R)}_{\chi'_R,\chi''_R}$), and operator expressions ($\hat{O}^{(R)}_{\chi''_R}$) to the Goldstone diagrams shown below. Write the complete algebraic expressions for each of the diagrams. We use the notation, in which a, i represent fixed indices, while c, d, e, k, l, m, n are free (summation) labels. The rightmost vertex in diagram (a) should be interpreted as the single excitation operator $\hat{C}_1$.

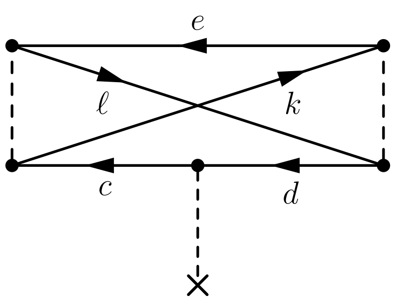
(a)



(b)



(c)



2. (4 pts.) Use the diagrammatic method based on drawing the resulting Goldstone diagrams to calculate

   (a) $\langle\Phi|V_N Z_N V_N|\Phi\rangle$,

   (b) $V_N|\Phi_i^a\rangle$.

   It is strongly recommended that you use the non-oriented and oriented Hugenholtz skeletons/diagrams in the initial steps. Remember, however, that the final product of your work should be all admissible non-equivalent resulting Goldstone diagrams and the corresponding algebraic expressions.

3. (4 pts.) Use the diagrammatic method to rederive Eq. (6) from Assignment No. 3.

4. (4 pts.) Use the diagrammatic arguments based on drawing the relevant non-oriented Hugenholtz skeletons to explain why matrix element $\langle\Phi_{i_1\ldots i_n}^{a_1\ldots a_n}|\,[Z_N,T_1]\,|\Phi\rangle$, where Z_N and T_1 are the one-body operators defined by Eqs. (4) and (5) in Assignment No. 3, vanishes when $n \geq 2$ (see Problem 3 in Assignment No. 3). As usual, $[,]$ is the commutator, $|\Phi\rangle$ is the Fermi vacuum state, and $|\Phi_{i_1\ldots i_n}^{a_1\ldots a_n}\rangle$ is the n-tuply excited determinant relative to $|\Phi\rangle$. Use a similar simplified diagrammatic analysis to determine the range of n values for which $\langle\Phi_{i_1\ldots i_n}^{a_1\ldots a_n}|\,[V_N,T_1]\,|\Phi\rangle$ is nonzero, where V_N is a two-body operator in the normal-ordered form. In providing your answer, you can use a convention in which the $n = 0$ determinant is equivalent to $|\Phi\rangle$ (no excitations).

5. (3 pts.) Draw the Hugenholtz diagrams that would correspond to the Goldstone diagrams (a)–(c) in Problem 1. Write the complete algebraic expressions for each of the resulting Hugenholtz diagrams. In reading the Hugenholtz diagrams, use the same notation as in Problem 1. Remember that the rightmost vertex in diagram (a) should be interpreted as the single excitation operator $\hat{C}_1$.

6. (2 pts.) Redo Problem 2 using the diagrammatic method based on the Hugenholtz-Brandow diagrams.