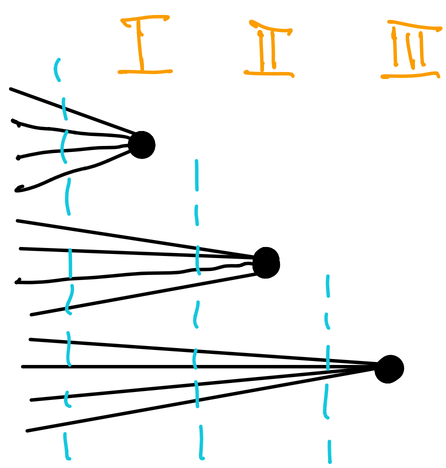
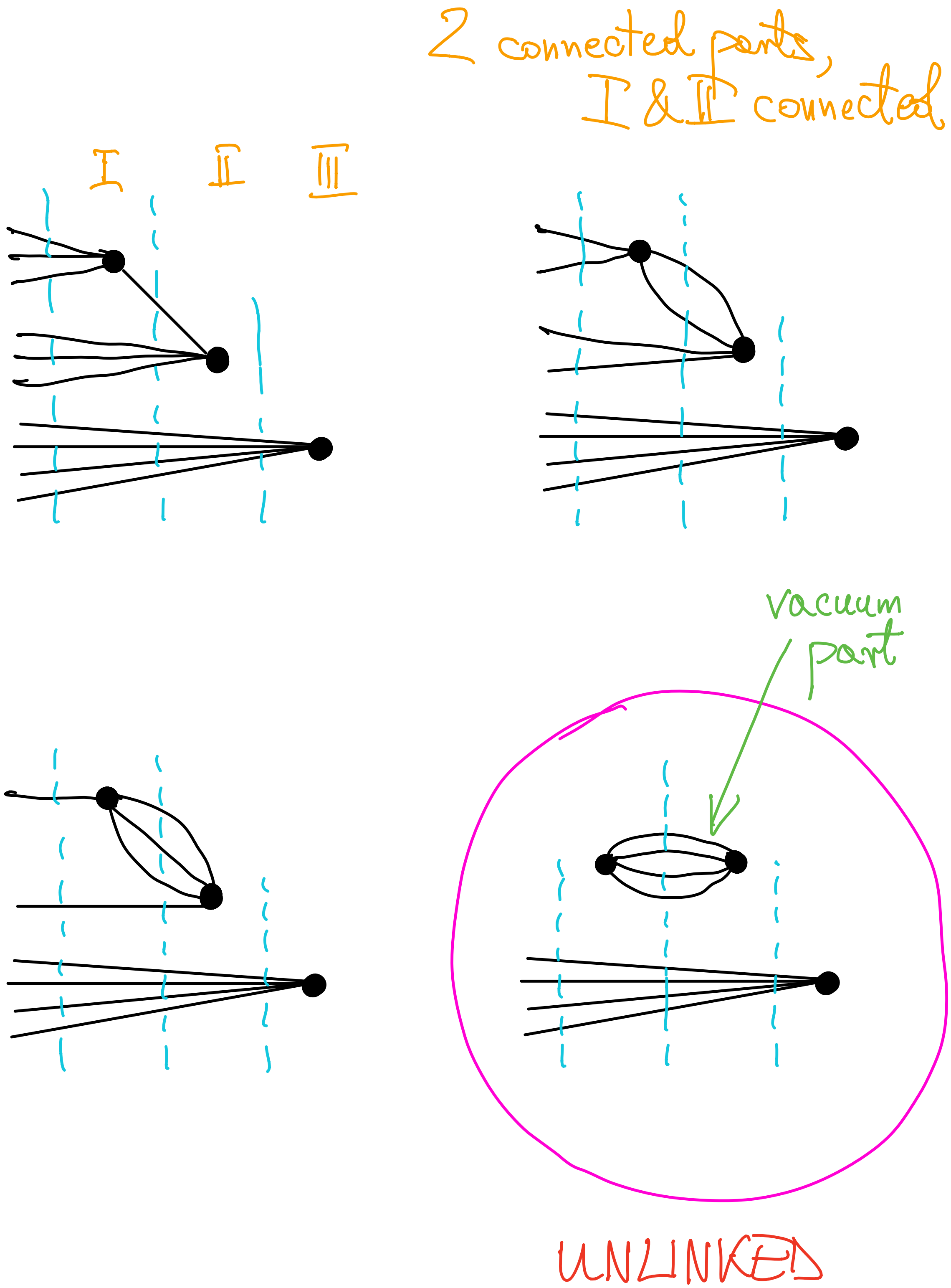
WAVE FUNCTION CORRECTIONS CONTINUED

$$|\Psi_0^{(3)}\rangle = R^{(0)} W R^{(0)} W R^{(0)} W |\Phi_0\rangle$$
$$- \underbrace{\langle\Phi_0|W R^{(0)} W|\Phi_0\rangle}_{k_0^{(2)}} R^{(0)2} W |\Phi_0\rangle$$

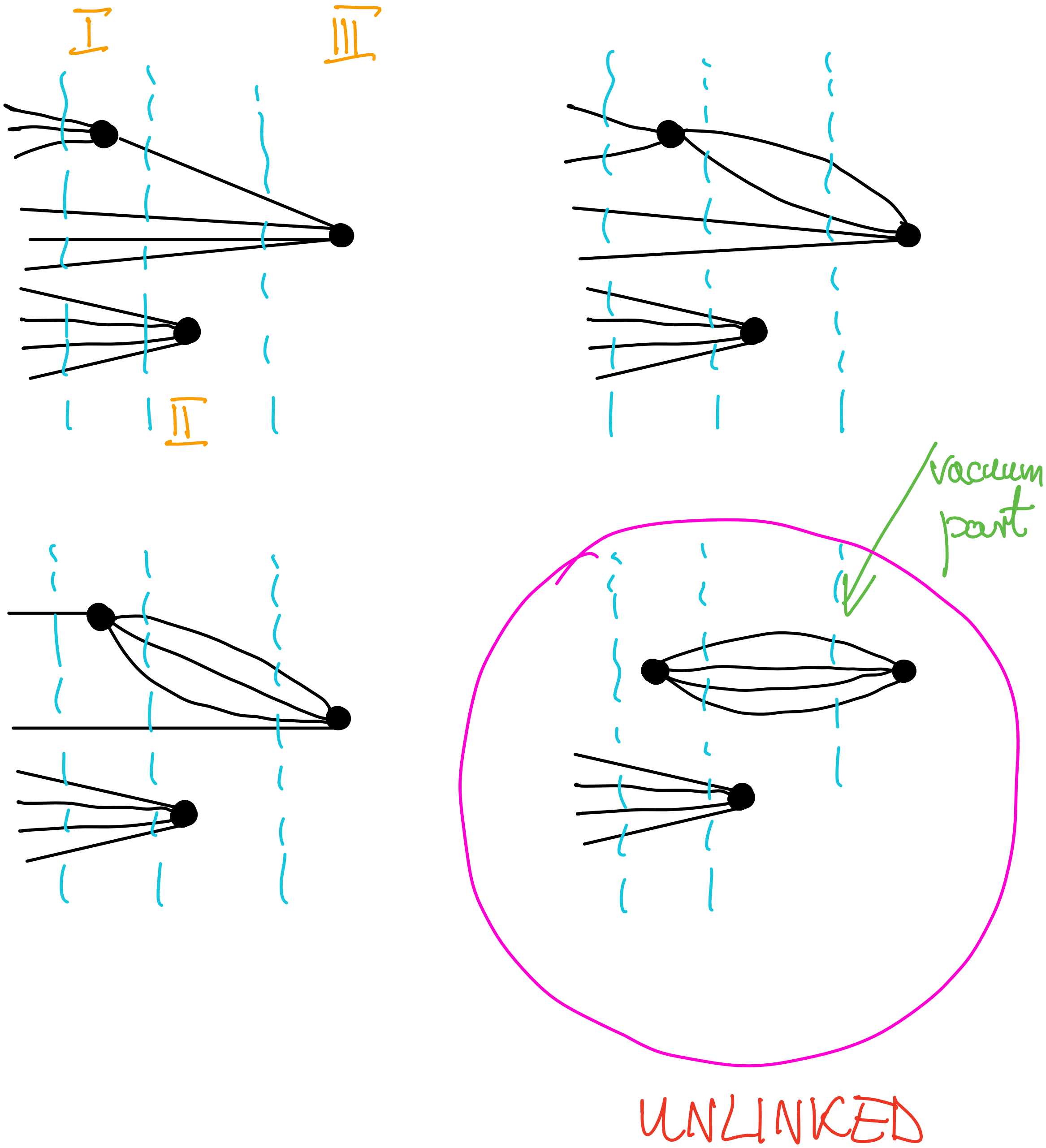
Let us examine the principal term assuming $W = V_N$.



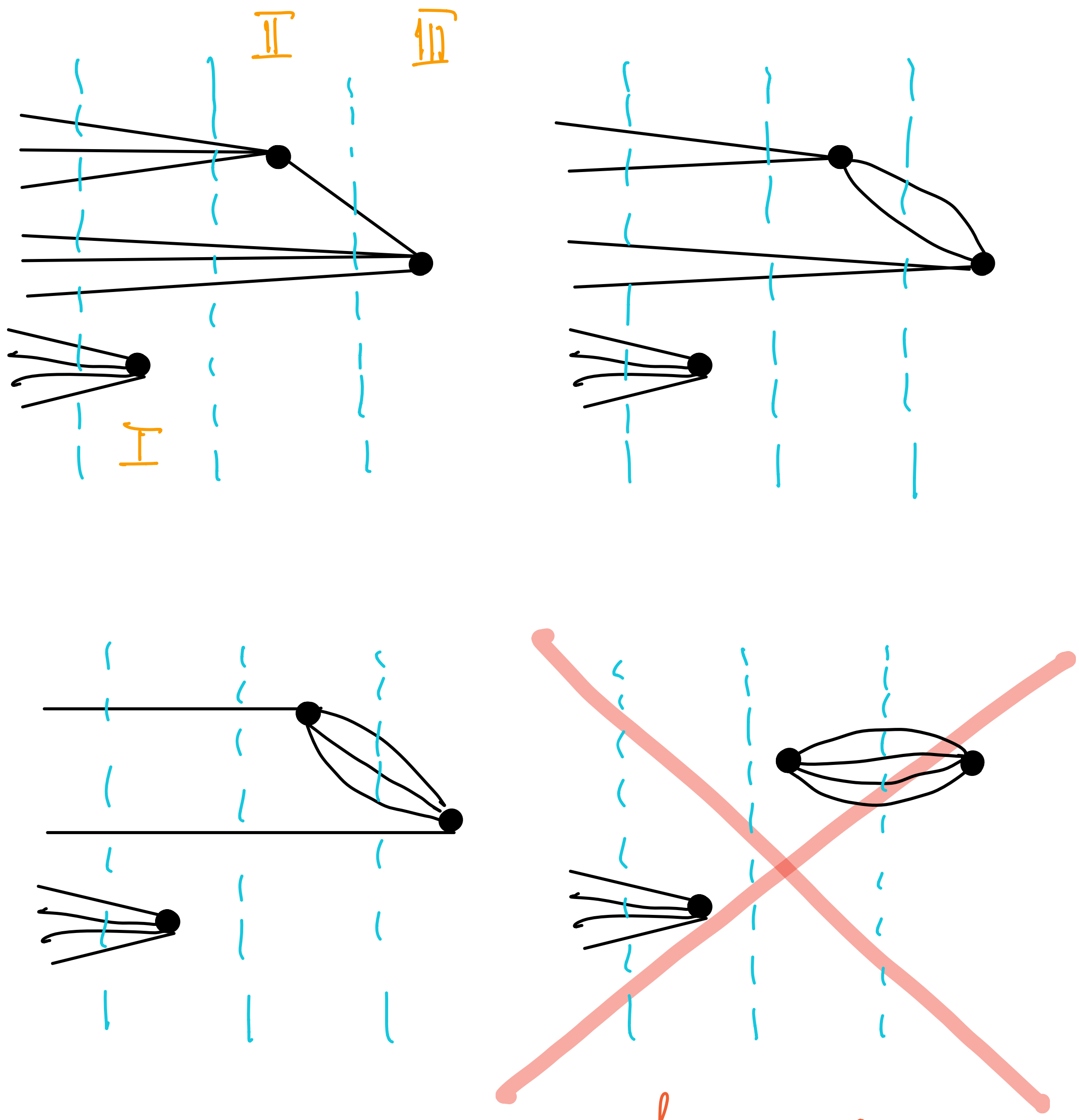
3 connected parts


2 connected parts,
I & II connected
I
II
III
vacuum part
UNLINKED

2 connected parts,
I & III connected

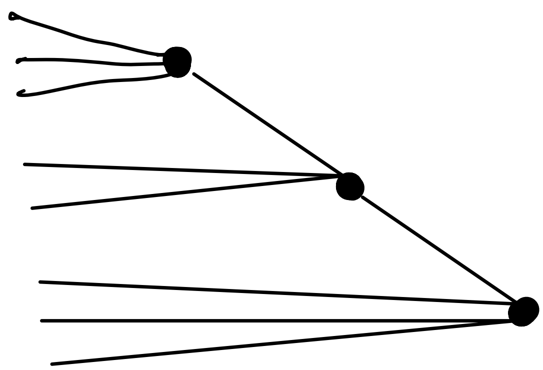


2 connected parts,
II & III connected



dangerous
denominator

Connected diagrams



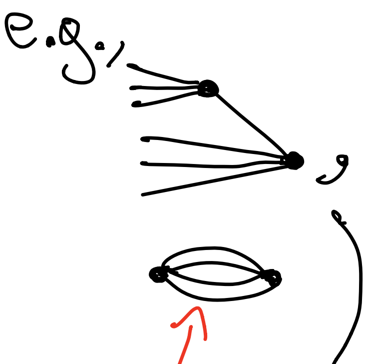
+ a lot more

The above disconnected diagrams containing vacuum parts are examples of

UNLINKED DIAGRAMS.

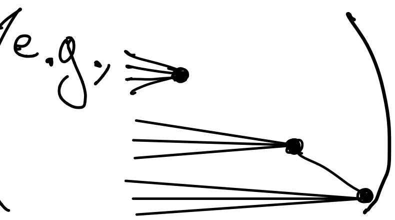
In general, a diagram is called unlinked if it is disconnected AND contains one or more vacuum components.

CONNECTED $\longrightarrow$ LINKED

(e.g., )

linked with no external lines

DISCONNECTED $\longrightarrow$ LINKED, UNLINKED

LINKED (e.g., )

UNLINKED (e.g., )

vacuum component

CONNECTED $\longrightarrow$ LINKED

DISCONNECTED $\longrightarrow$ LINKED

DISCONNECTED $\longrightarrow$ UNLINKED

$$|\Psi_0^{(3)}\rangle = \Big[\{(R^{(0)}W)^3\}_{C}\, |\Phi_0\rangle$$

connected

$$+ \{(R^{(0)}W)^3\}_{DC,L}\, |\Phi_0\rangle$$

disconnected linked

$$+ \{(R^{(0)}W)^3\}_{UL}\, |\Phi_0\rangle \Big]$$

unlinked

principal term $[\,\cdots]$

$$- \langle\Phi_0|WR^{(0)}W|\Phi_0\rangle\, R^{(0)2}W\, |\Phi_0\rangle$$

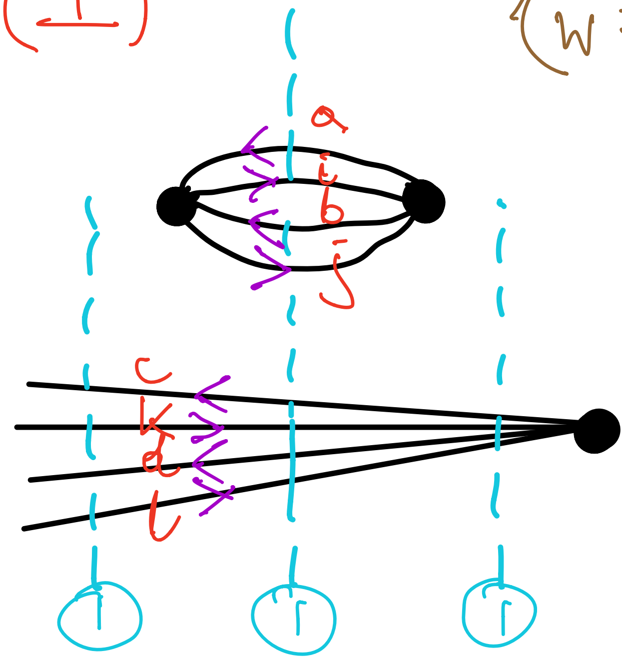
renormalization term

Let us focus on unlinked diagrams,

$$\{(R^{(0)}W)^3\}_{UL}\, |\Phi_0\rangle .$$

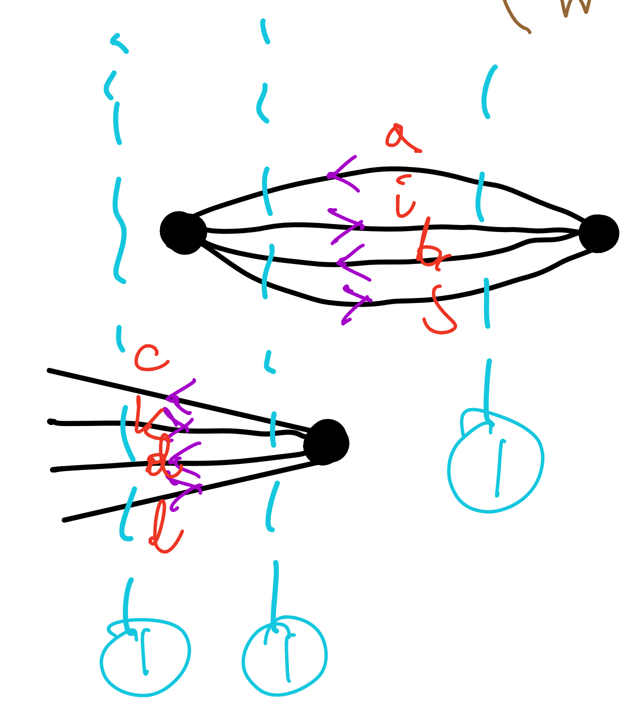
(I)

$\left(w = \frac{1}{16}\right)$

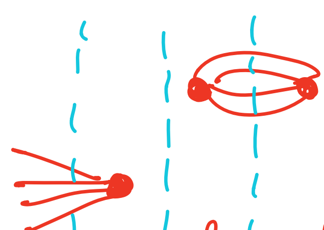


(II)

$\left(w = \frac{1}{16}\right)$



These two diagrams are all time versions of the first kind (the third time version,



is disallowed by the denominator rule)

$$a = \varepsilon_i + \varepsilon_j - \varepsilon_a - \varepsilon_b$$

$$b = \varepsilon_k + \varepsilon_l - \varepsilon_c - \varepsilon_d$$

Weights, signs, and matrix elements corresponding to (I) and (II) are identical.

N $\rightarrow$ numerator in (I) and (II).

$$(\mathrm{I}): \quad \frac{N}{b(a+b)\,b}$$

$$(\mathrm{II}): \quad \frac{N}{b(a+b)\,a}$$

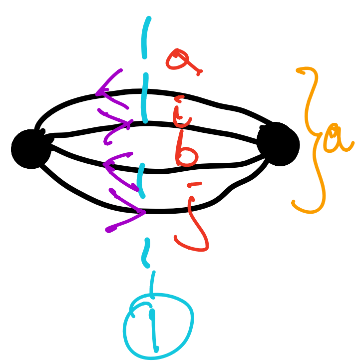
$$\{(R^{(0)}W)^3\}_{w_L}|\Phi_0\rangle$$

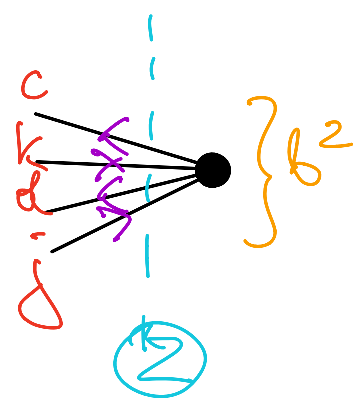
$$= (\mathrm{I}) + (\mathrm{II})$$

$$= \frac{N}{b(a+b)}\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$= \frac{N}{b\cancel{(a+b)}}\,\frac{\cancel{(a+b)}}{ab}$$

$$= \frac{N}{ab^2}$$

$=$ 

$\times$ 

$$= \langle \Phi_0 | W R^{(0)} W | \Phi_0 \rangle \, R^{(0)2} W | \Phi_0 \rangle$$

(negative of the renormalization term)

$$| \Psi_0^{(3)} \rangle = \Big[\{ (R^{(0)} W)^3 \}_C | \Phi_0 \rangle$$

$$+ \{ (R^{(0)} W)^3 \}_{DC,L} | \Phi_0 \rangle$$

$$\cancel{+ \{ (R^{(0)} W)^3 \}_{UL} | \Phi_0 \rangle} \Big]$$

$$\cancel{- \langle \Phi_0 | W R^{(0)} W | \Phi_0 \rangle \, R^{(0)2} W | \Phi_0 \rangle}$$

$$|\Psi_0^{(3)}\rangle = \{(R^{(0)}W)^3\}_L |\Phi_0\rangle$$

THIS TYPE OF CANCELLATION HAPPENS IN ALL MBPT WAVE FUNCTION ORDERS,

$$|\Psi_0^{(n)}\rangle = \{(R^{(0)}W)^n\}_L |\Phi_0\rangle$$

$(n=1,2,\dots)$

LINKED CLUSTER THEOREM

More examples $\left(\{(R^{(2)}W)^4\}_{\mathcal{U}}, |\Psi_0\rangle\right)$ in the lecture videos and notes.

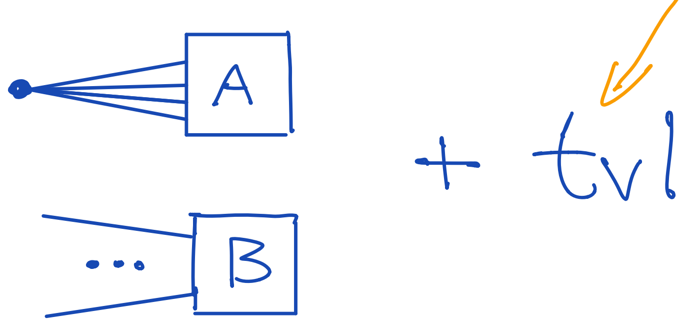
The next major topic, after discussing the significance of QEPV diagrams, is the

FACTORIZATION LEMMA.

(L.M. Frantz, R.L. Mills, Nucl. Phys. 15, 16 (1960))

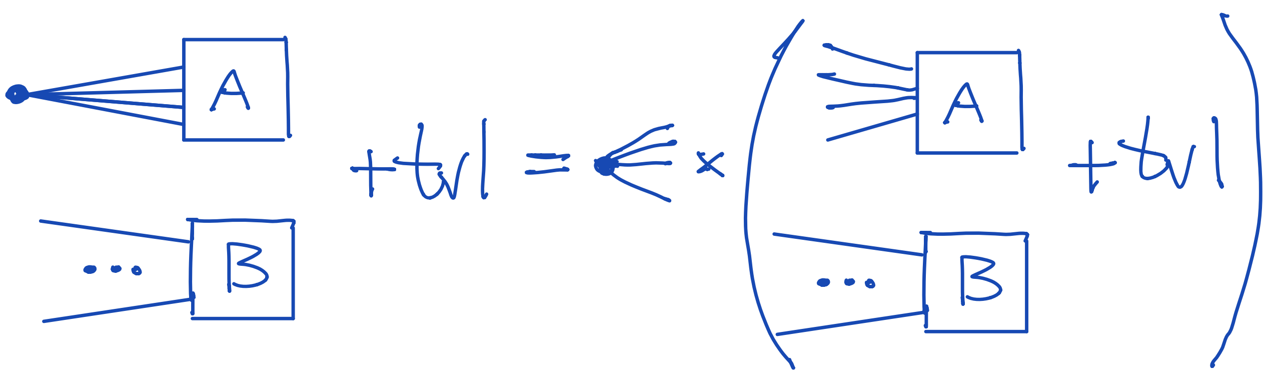
We need it, since during the proof of the linked cluster theorem, we will encounter situations, such as

time versions of the 1st kind

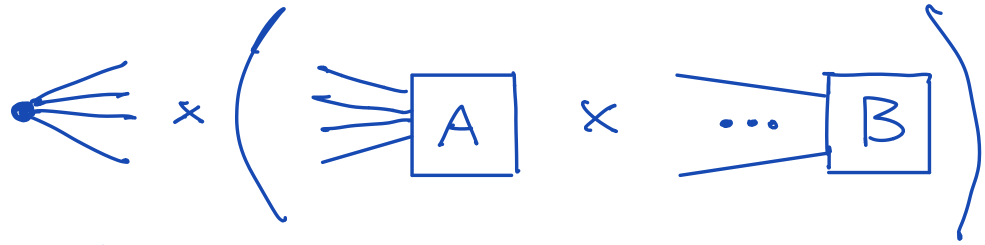


(unlinked diagrams with external lines)

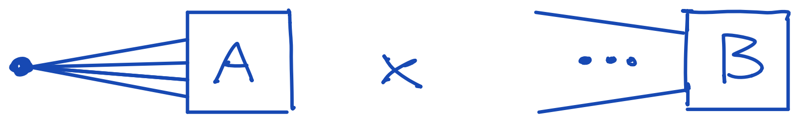
We will do the following:



FL

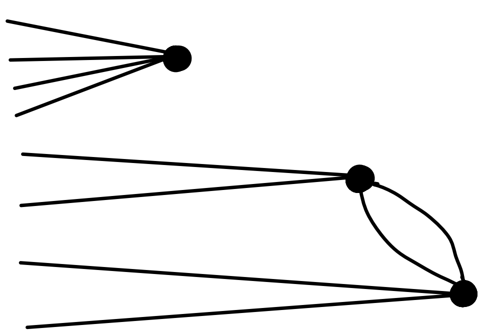
$=$ 

factorization lemma

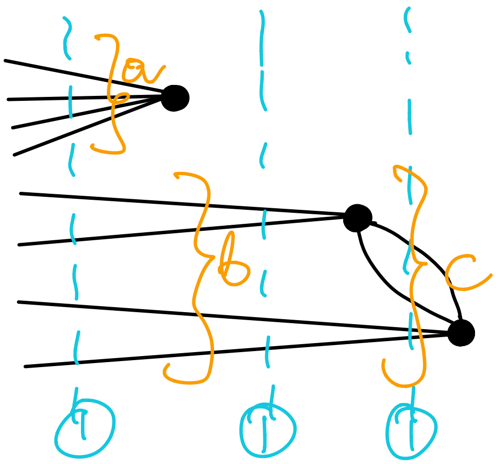
$=$  .

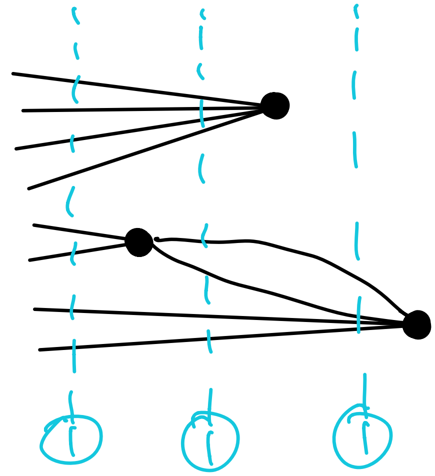
In the factorization lemma, we deal with disconnected, but linked diagrams and their time versions of the 1st kind.

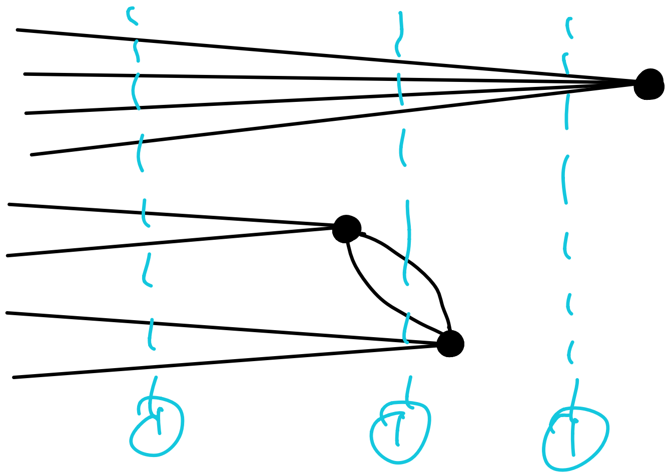
Examples:



+ all nonequivalent time versions of the 1st kind

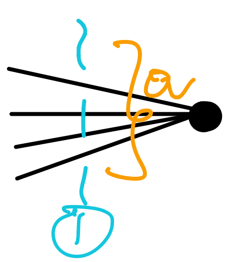
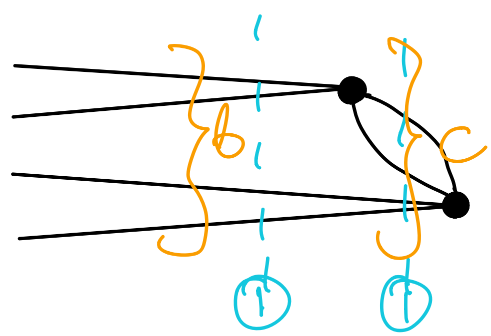






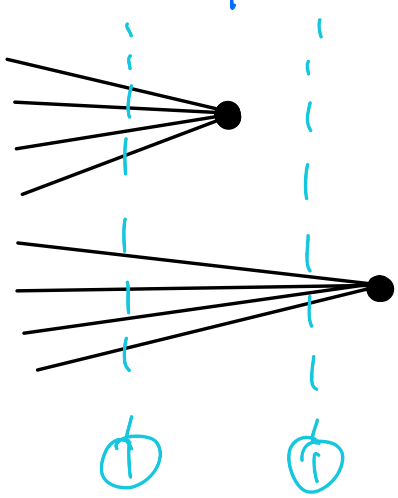
$$\frac{N}{(a+b)\,b\,c} + \frac{N}{(a+b)(a+c)\,c} + \frac{N}{(a+b)(a+c)\,a}$$

$$= \frac{N}{a\,b\,c}$$

$=$  $\times$ 

In the above example, the two connected components are different. What about cases where the two (or more) connected components are equivalent?

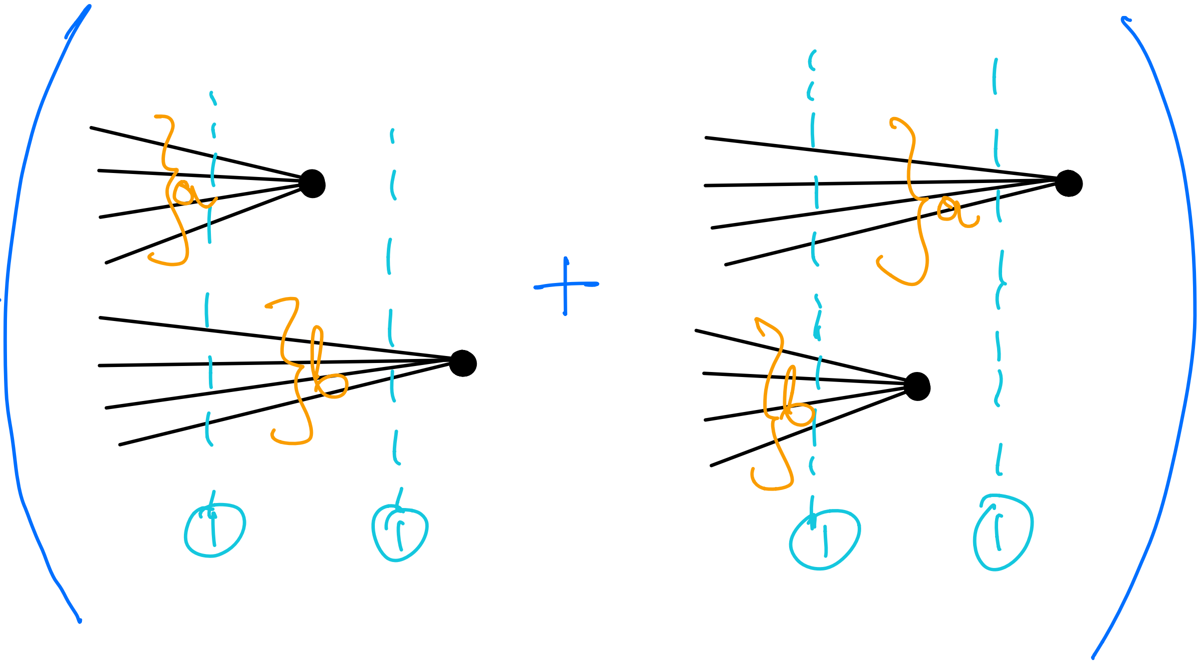
Example:



In this case, there is only one time version of the 1st kind.

Two achieve factorization, we write the above d-m twice and divide the result by 2.

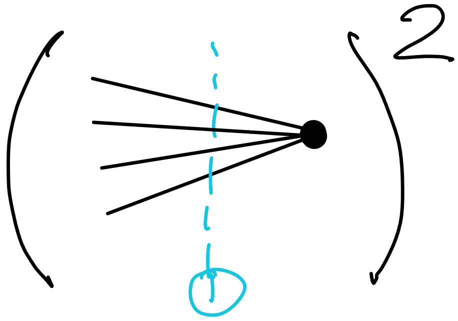
$\frac{1}{2}$



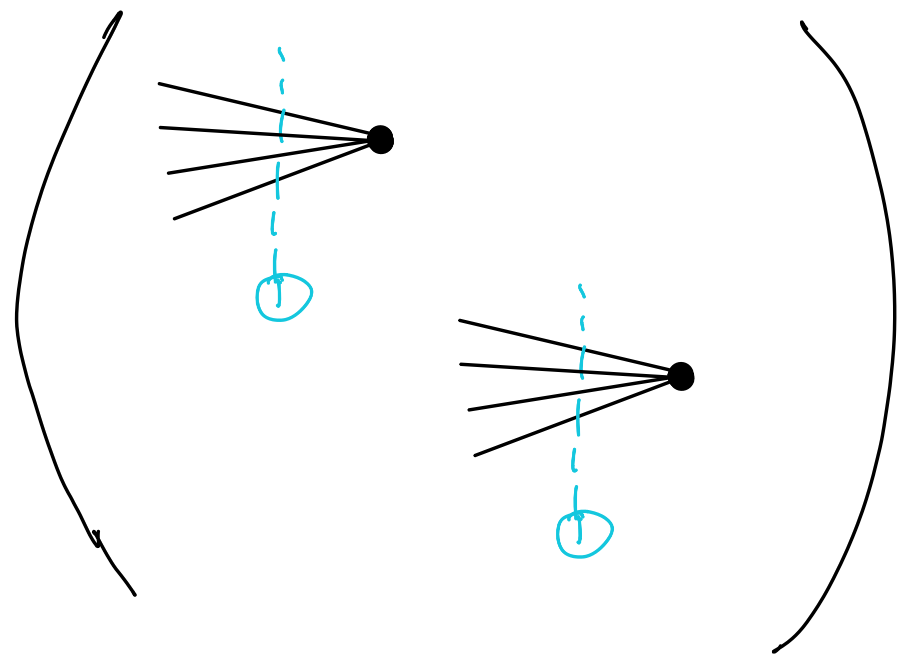
$$= \frac{1}{2}\left(\frac{N}{(a+b)\,b} + \frac{N}{(a+b)\,a}\right)$$

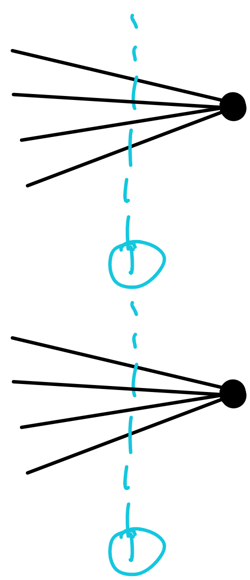
$$= \frac{1}{2}\,\frac{N}{ab}$$

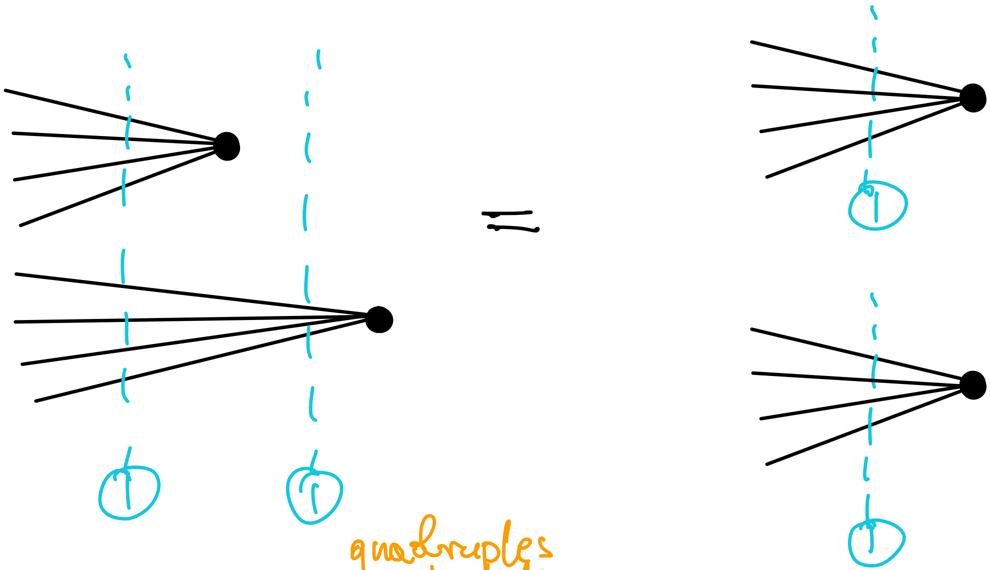
$= \frac{1}{2}$



Consistent with our diagrammatic rules,

$\frac{1}{2}$ 

$=$ 



quadruples contribution

$$|\Psi_0^{(2)}(Q)\rangle = \frac{1}{2}\left(R^{(0)}W\right)^2|\Phi_0\rangle$$

$$= \frac{1}{2}\left(T_2^{(1)}\right)^2|\Phi_0\rangle$$

k-body component of T

$$T_k|\Phi_0\rangle = \sum_n \left\{\left(R^{(0)}W\right)^n\right\}_{C,k}|\Phi_0\rangle$$

k-body (2k external lines)

cluster operator

$$T = \sum_{k=1}^{N} T_k$$

$$\sum_{n=0}^{\infty}|\Psi_0^{(n)}\rangle = e^T|\Phi_0\rangle$$

For more examples, see lecture notes/videos.

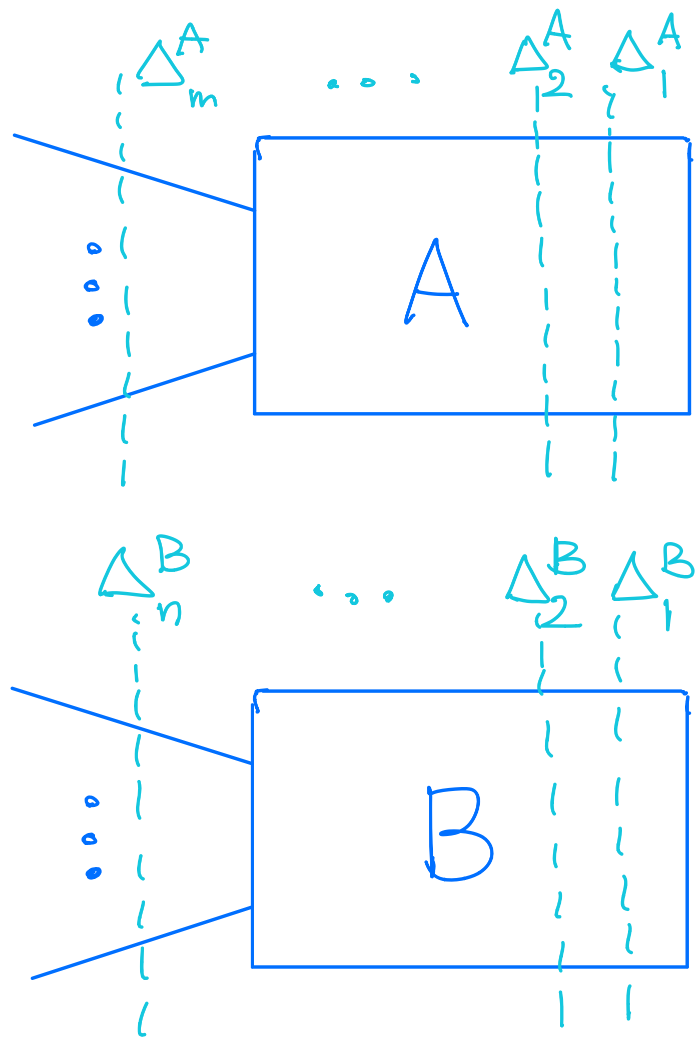
Factorization lemma:

Consider all possible time versions of the first kind for a LINKED DISCONNECTED diagram consisting of two parts A and B. These two parts are not necessarily connected, so the lemma applies to all linked disconnected diagrams. We designate the set of denominators for part A as

$$\Delta_\mu^A, \mu = 1, \ldots, m$$

and for part B as

$$\Delta^B_\nu, \quad \nu = 1, \ldots, n.$$



Let us define

$$D_m^A = \prod_{\mu=1}^{m} (\Delta_\mu^A)^{-1},$$

$$D_n^B = \prod_{\nu=1}^{n} (\Delta_\nu^B)^{-1}.$$

The denominator contribution from all possible time versions of the first kind corresponding to all possible orderings of vertices in parts A and B relative to one another,

$$D_{mn}^{AB},$$

equals

$$D_m^A \, D_n^B.$$

Proof in video 39 and pages 596 - 605.

LINKED CLUSTER THEOREM

(video 40)

$$|\Psi_0^{(n)}\rangle = \left\{ (R^{(0)}W)^n \right\}_L |\Phi_0\rangle,$$

$n = 1, 2, \dots$

linked diagrams [originating from $(R^{(0)}W)^n$ including EPV terms]

The above formula implies that

$$K_0^{(n+1)} = \langle \Phi_0 | \{ W (R^{(0)} W)^n \}_{C_0} | \Phi_0 \rangle .$$

connected, no external lines

Here is why:

$$K_0^{(n+1)} = \langle \Phi_0 | W | \Psi_0^{(n)} \rangle \quad \leftarrow \text{from RSPT}$$

$$= \langle \Phi_0 | W \underbrace{\{ (R^{(0)} W)^n \}_L}_{\text{no vacuum parts}} | \Phi_0 \rangle$$

$$= \langle \Phi_0 | \{ W \{ (R^{(0)} W)^n \}_L \}_{C_0} | \Phi_0 \rangle$$

We can drop "L", since the

$\{(R^{(0)}W)^n\}_{UL}$ part of $(R^{(0)}W)^n$ does not contribute (we would be left with disconnected contributions to $k_0^{(n+1)}$ due to vacuum pieces in $\{(R^{(0)}W)^n\}_{UL}$).

We obtain,

$$k_0^{(n+1)} = \langle \Phi_0 | \{ W \, (R^{(0)} W)^n \}_{C_0} | \Phi_0 \rangle .$$

This means that

$$|\Psi_0\rangle = \sum_{n=0}^{\infty} |\Psi_0^{(n)}\rangle = \sum_{n=0}^{\infty} \{(R^{(0)}W)^n\}_L |\Phi_0\rangle$$

and

$$k_0 = \varepsilon_0 + \sum_{n=0}^{\infty} k_0^{(n+1)}$$

$$= \varepsilon_0 + \sum_{n=0}^{\infty} \langle \Phi_0 | \{ W (R^{(0)} W)^n \}_{C_0} | \Phi_0 \rangle$$

should satisfy the Schrödinger equation,

$$(K_0+W)|\Psi_0\rangle = k_0|\Psi_0\rangle.$$

This is exactly how we will prove the linked cluster theorem.

Proof (initial stages):

$$(\varkappa_0-K_0)|\Psi_0\rangle$$

$$=(\varkappa_0-K_0)\left[|\Phi_0\rangle+\sum_{n=1}^{\infty}\{(R^{(0)}W)^n\}_L|\Phi_0\rangle\right]$$

$$=(\varkappa_0-K_0)\sum_{n=1}^{\infty}\{(R^{(0)}W)^n\}_L|\Phi_0\rangle$$

$$=(\varkappa_0-K_0)R^{(0)}\sum_{n=0}^{\infty}\{W(R^{(0)}W)^n\}_{L_{\text{ext}}}|\Phi_0\rangle$$

because $R^{(0)}$ has external lines, linked with external lines

$$(\mathcal{E}_0 - K_0) R^{(0)} = Q = 1 - |\Phi_0\rangle\langle\Phi_0| \quad !$$

$$
\begin{aligned}
(\mathcal{E}_0 - K_0)|\Psi_0\rangle
&= \left(1 - |\Phi_0\rangle\langle\Phi_0|\right) \sum_{n=0}^{\infty} \left\{W\left(R^{(0)}W\right)^n\right\}_{L_{ext}} |\Phi_0\rangle \\
&= \sum_{n=0}^{\infty} \left\{W\left(R^{(0)}W\right)^n\right\}_{L_{ext}} |\Phi_0\rangle \\
&\quad - |\Phi_0\rangle \sum_{n=0}^{\infty} \underbrace{\langle\Phi_0| \left\{W\left(R^{(0)}W\right)^n\right\}_{L_{ext}} |\Phi_0\rangle}_{=\,0}
\end{aligned}
$$

(L_{ext} has external lines)

$$(\mathcal{E}_0 - K_0)|\Psi_0\rangle$$

$$= \sum_{n=0}^{\infty} \left\{ W (R^{(0)} W)^n \right\}_{L_{ext}} |\Phi_0\rangle$$

$$\{ \qquad \}_L$$

$$= \sum_{n=0}^{\infty} \left\{ W \left\{ (R^{(0)} W)^n \right\}_L \right\}_{L_{ext}} |\Phi_0\rangle$$

linked

linked with external lines

This is because adding W to the UL diagram cannot produce a linked one.

In order to continue, we must learn what happens when one applies W to

$$\sum_{n=0}^{\infty}\left\{\left(R^{(0)}W\right)^n\right\}_L |\Phi_0\rangle .$$

We will calculate

$$W \underbrace{\sum_{n=0}^{\infty}\left\{\left(R^{(0)}W\right)^n\right\}_L |\Phi_0\rangle}_{|\Psi_0\rangle}$$

NEXT TIME