INTRODUCTION TO THE SINGLE-REFERENCE MANY-BODY PERTURBATION THEORY AND ITS DIAGRAMMATIC REPRESENTATION

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Department of Chemistry and Department of Physics & Astronomy, Michigan State University, East Lansing, Michigan 48824, USA

SHORT COURSE OFFERED DURING THE XINGDA LECTURESHIP VISIT

COLLEGE OF CHEMISTRY AND MOLECULAR ENGINEERING, PEKING UNIVERSITY, BEIJING, CHINA, NOVEMBER 12-14, 2019

MANY THANKS TO PROFESSOR KAI WU AND COMMITTEE FOR ACADEMIC EXCHANGES FOR INVITATION AND PROFESSOR JIAN LIU FOR HOSPITALITY
MANY-PARTICLE SCHRÖDINGER EQUATION

QUANTUM CHEMISTRY: THE ELECTRONIC SCHRÖDINGER EQUATION

\[ H_e \Psi_K(X; R) = E_K(R) \Psi_K(X; R) \]

\[ H_e = Z + V = \sum_{i=1}^{N} z(x_i) + \sum_{i>j=1}^{N} v(x_i, x_j) \]

\[ z(x_i) = -\frac{1}{2} \Delta_i + \sum_{A=1}^{M} \frac{Z_A}{R_{Ai}}, \quad v(x_i, x_j) = \frac{1}{r_{ij}} \]
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NUCLEAR PHYSICS: THE NUCLEAR SCHRÖDINGER EQUATION

\[ H_n \Psi_\mu(X) = E_\mu \Psi_\mu(X) \]

\[ H_n = Z + V_2 + V_3(???) = \sum_{i=1}^{N} z(x_i) + \sum_{i>j=1}^{N} v_2(x_i, x_j) + \sum_{i>j>k=1}^{N} v_3(x_i, x_j, x_k)(???) \]

\[ z(x_i) = \frac{p_i^2}{2m_i}, \quad v_2(x_i, x_j) = ? (\text{Argonne } v_{18}, \text{CD Bonn, Idaho-A, etc.}), \]

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MANY-BODY TECHNIQUES DEVELOPED IN ONE AREA SHOULD BE APPLICABLE TO OTHER AREAS

or NLO, N^2LO, N^3LO, etc.
SOLVING THE MANY-PARTICLE SCHRÖDINGER EQUATION

- Define a basis set of single-particle functions (e.g., LCAO-type molecular spin-orbitals in quantum chemistry obtained by solving mean-field equations or harmonic oscillator basis in nuclear physics)

\[ V \equiv \{ \varphi_r(x), r = 1, \ldots, \dim V \} \]

Exact case: \( \dim V = \infty \), in practice: \( \dim V < \infty \)

- Construct all possible Slater determinants that can be formed from these spin-particle states

\[
\Phi_{r_1 \ldots r_N}(x_1, \ldots, x_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix}
\varphi_{r_1}(x_1) & \cdots & \varphi_{r_1}(x_N) \\
\vdots & \ddots & \vdots \\
\varphi_{r_N}(x_1) & \cdots & \varphi_{r_N}(x_N)
\end{vmatrix}
\]
The exact wave function can be written as a linear combination of all Slater determinants

$$\Psi_\mu(x_1, \ldots, x_N) = \sum_{r_1<\cdots<r_N} c_{r_1\cdots r_N}^\mu \Phi_{r_1\cdots r_N}(x_1, \ldots, x_N)$$

$$= \sum_I c_I^\mu \Phi_I(x_1, \ldots, x_N)$$

Determine the coefficients $c$ and the energies $E_\mu$ by solving the matrix eigenvalue problem:

$$HC^\mu = E_\mu C^\mu$$

where the matrix elements of the Hamiltonian are

$$H_{IJ} = \langle \Phi_I \mid \hat{H} \mid \Phi_J \rangle = \int dx_1 \cdots dx_N \Phi_I^*(x_1, \ldots, x_N) \hat{H} \Phi_J(x_1, \ldots, x_N)$$

This procedure, referred to as the full configuration interaction approach (FCI), yields the exact solution within a given single-particle basis set.
THE PROBLEM WITH FCI
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Dimensions of the full CI spaces for many-electron systems

<table>
<thead>
<tr>
<th>Orbitals</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
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<tbody>
<tr>
<td>20</td>
<td>$3.79 \times 10^3$</td>
<td>$5.80 \times 10^6$</td>
<td>$52.6 \times 10^6$</td>
<td>$300 \times 10^6$</td>
</tr>
<tr>
<td>30</td>
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Dimensions of the full shell model spaces for nuclei

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Alternative approaches are needed in order to study the majority of many-body systems of interest.
The key to successful description of atoms, molecules, condensed matter systems, and nuclei is an accurate determination of the MANY-PARTICLE CORRELATION EFFECTS. INDEPENDENT-PARTICLE-MODEL APPROXIMATIONS, such as the Hartree-Fock method, ARE USUALLY INADEQUATE.
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This is a short course on single-reference MBPT aimed at the following content:

1. Preliminaries: molecular electronic Schrödinger equation, Slater determinants, CI wave function expansions, and elements of second quantization.
2. Rayleigh-Schrödinger perturbation theory, wave, reaction, and reduced resolvent operators.
3. Eigenfunction and eigenvalue expansions, renormalization terms, and bracketing technique.
5. MBPT diagrams in low orders (second-, third-, and fourth-order energy corrections; first- and second-order wave function contributions).
6. Linked, unlinked, connected, and disconnected diagrams; diagram cancellations in fourth-order energy and third-order wave function corrections.
7. Linked and connected cluster theorem and their implications.
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Time permitting, we will expand on point 7 and discuss basic elements of the coupled-cluster theory.
DESCRIPTION OF MANY-PARTICLE CORRELATION EFFECTS BY SINGLE-REFERENCE MANY-BODY PERTURBATION THEORY (MBPT)

This will be a short course on single-reference MBPT based on the following materials:

1. Lecture notes that will be provided to you in a PDF format.


3. Lecture notes by Professor Josef Paldus, which can be downloaded from www.math.uwaterloo.ca/~paldus/resources.html.
DESCRIPTION OF MANY-PARTICLE CORRELATION EFFECTS BY SINGLE-REFERENCE MANY-BODY PERTURBATION THEORY (MBPT)

Although the use of perturbation theory to analyze the many-electron correlation problem dates back to the seminal 1934 work by Møller and Plesset, the Møller and Plesset work is limited to the second order and does not use second quantization.
DESCRIPTION OF MANY-PARTICLE CORRELATION EFFECTS BY SINGLE-REFERENCE MANY-BODY PERTURBATION THEORY (MBPT)

They key original papers most relevant to this presentation of MBPT are:

after these corrections and that for compound nucleus contribution a similar discrepancy yet remains, it is in the sense to correspond to a greater reduced width for neutrons in C^9 than for protons in N^9; i.e., “the neutrons stick out further than the protons.” Such an effect has been suggested for heavier nuclei, though it would be very surprising to find it holding for so light a nucleus as A = 18.

An estimate of the course of the cross section for the reaction C^9(d,p)C^8 was made on the basis of compound nucleus fission by assuming, as before, that the whole of the cross section for C^9(d,p)C^8 at low deuteron energies involves compound nucleus formation. On the assumption that the reduced width for triton emission is as great as that for neutron emission (the assumption of “preferred” tritons), we predict the dashed line of Fig. 2—in which the coming into play of successive residual states of C^8 has been allowed for and the associated irregularities smoothed out. It is seen that even under the very unphysical assumption of the existence of preferred tritons, compound nucleus fission by an order of magnitude to explain the observed C^8 formation. We are forced then to assume that this (d,p) reaction proceeds by some pickup mechanism and that we are indeed measuring the relative probability of the deuteron’s losing a nucleon to the nucleus and removing one from it. As yet no sufficiently reliable theory of (d,p) pickup exists to warrant a comparison being made with these results. It is interesting to note that, at E_d = 3.3 Mev, the angular distribution of the reaction C^9(d,p)C^8 is such as to suggest that a direct mechanism already predominates.

It is interesting to compare these results with those of Cohen and Handy in (p,n) reactions. These authors suggest that triton emission from a compound nucleus state has an inherent probability comparable with that for single nucleon emission. They base this argument on the rather flat angular distributions sometimes obtained which, they remark, tell against a pickup process. However, this conclusion is no longer valid when the energy of one or both the charged particles concerned is of the order of or below the Coulomb barrier; here a direct mechanism can give a sensibly isotropic angular distribution. It appears that considerable interest attaches to the resolution of this question of the mechanism by which tritons and similar complicated particles are emitted from nuclei in events of moderate to high energy.

D. H. WILKINSON

Many-Body Problem for Strongly Interacting Particles. II. Linked Cluster Expansion

K. A. Brueckner
Indiana University, Bloomington, Indiana
Received April 20, 1955

An approximation method developed previously to deal with many particles in strong interaction is examined in further detail. It is shown that the series giving the interaction energy is a development of a sequence of linked or additive cluster terms each of which gives a contribution to the energy proportional to the total number of particles. Consequently the convergence of the expansion is independent of the total number of particles. The origin of this aspect is illustrated by showing that a similar situation exists in the expansion of a short-range perturbation theory. The physical meaning of the expansion is quasi-quantitatively discussed for the nuclear problem where it is shown that the correction arising from the first cluster term involving three particles is less than the leading term by a factor of about 10^{−3}. The smallness of the correction is largely a result of the action of the exclusion principle.

I. INTRODUCTION

In a previous paper (to be referred to as I) we have given a method for reducing approximately the many-body problem for strongly interacting particles to a problem of self-consistent fields. Some of the physical concept and origin of the method were discussed there and the nature of certain correction terms which can be neglected for very many particles was discussed. We shall in this paper examine the structure of another type of correction term which arises from interaction

of clusters of particles and in so doing exhibit the general structure of the expansion involved. This will also allow us to draw some general conclusions about the convergence and accuracy of the method.

In Sec. II, we shall briefly summarize the relevant formulas from I and describe some difficulties which appear in high-order terms in the expansion for the energy which can be removed by a simple modification of the many-body propagation function. In Sec. III, we show how similar terms appear to arise in the usual perturbation theory but that they cancel identically, in a manner simply related to the cancellation discussed in Sec. II. In Sec. IV, we summarize these results and show how they may be generalized into a simple pre-
DESCRIPTION OF MANY-PARTICLE CORRELATION EFFECTS BY SINGLE-REFERENCE MANY-BODY PERTURBATION THEORY (MBPT)

They key original papers most relevant to this presentation of MBPT are:

Derivation of the Brueckner many-body theory

BY J. Goldstone

Trinity College, University of Cambridge

(Communicated by N. F. Mott, P.R.S.—Received 24 August 1956)

An exact formal solution is obtained to the problem of a system of fermions in interaction. This solution is expressed in a form which avoids the problem of unlinked clusters in many-body theory. The technique of Feynman graphs is used to derive the series and to define linked terms. The graphs are those appropriate to a system of many fermions and are used to give a new derivation of the Hartree-Fock and Brueckner methods for this problem.

1. Introduction

The Hartree-Fock approximation for the many-body problem uses a wave function which is a determinant of single-particle wave functions—that is, an independent-particle model. The single-particle states are eigenstates of a particle in a potential \( V \), which is determined from the two-body interaction by a self-consistent calculation. The Brueckner theory (Brueckner & Levinson 1955; Bethe 1956; Eden 1956) gives an improved method of defining \( V \) and shows why the residual effects of \( V \) not allowed for by \( V \) can be small. In particular, in the nuclear problem the corrections to the energy are small, even though the corrections to the wave function are large. The theory thus gives a reconciliation of the shell model, the strong two-nucleon interactions, and the observed two-body correlations in the nucleus. The smallness of the corrections is due to the operation of the exclusion principle. Bethe (1956) has shown that this same exclusion effect makes even the Hartree-Fock approximation good for quite strong interactions, such as an exponential potential fitted to low-energy nucleon-nucleon scattering.

The first problem on which calculations have been made is that of ‘nuclear matter’, that is, a very large nucleus with surface effects neglected (Brueckner 1955; Wada & Brueckner 1956). In this problem the aim is to show that at a fixed density the energy is proportional to the number of particles, and that as the density is varied the energy per particle has a minimum at the observed density of large nuclei, and that this minimum value gives the observed volume energy of large nuclei. The single-particle wave functions are plane waves, and the potential \( V \) is diagonal in momentum space (in contrast to the ordinary Hartree potential which is diagonal in configuration space). The independent-particle model state is a ‘Fermi gas’ state with all the one-particle states filled up to the Fermi momentum \( k_F \) which depends only on the density.

Brueckner & Levinson’s derivation, and that of Eden, is based on the multiple scattering formalism of Watson (Watson 1955). The proportionality of the energy of nuclear matter of a given density to the number of particles follows at once from the theory provided certain terms which represent several interactions occurring
They key original papers most relevant to this presentation of MBPT are:

1. The description of collective motions in terms of many-body perturbation theory

By J. Hubbard

Atomic Energy Research Establishment, Harwell, Didcot, Berkshire

(Communicated by R. E. Peierls, F.R.S.—Received 2 February 1957)

In this and a succeeding paper it is shown how a theory equivalent to the Bohm & Pines collective motion theory of the electron plasma can be derived directly from a perturbation series which gives in principle an exact solution of the many-body problem. This result is obtained by making use of a diagrammatic method of analysis of the perturbation series. By a procedure analogous to the elimination of photon self-energy parts from the electromagnetic S-matrix it is found possible to simplify the perturbation series, introducing a modified interaction between the particles. A useful integral equation for this modified interaction can be set up, and it is shown how the energy of the system can be expressed in terms of the modified interaction. The close connection between this approach and the dielectric theory of plasma oscillations is indicated.

1. Introduction

Within recent years much attention has been given in the study of the quantum mechanical many-body problem to the collective motion of modes which may be present (Bohm & Pines 1953; Tomonaga 1955; Bohr & Mottelson 1953). Two main theories of collective motion have been developed, that of Tomonaga (1955), and the superfluous co-ordinate type of theory introduced by Bohm & Pines (1953). In the Tomonaga theory a transformation of variables is made in such a way that some of the new co-ordinates are directly related to the collective modes of motion, whilst the remaining new co-ordinates are associated with internal modes of motion. In the superfluous co-ordinate treatment certain auxiliary variables are introduced together with an equal number of subsidiary conditions to preserve the correct number of degrees of freedom, and a transformation is made in such a way that the new auxiliary variables are related to the collective motion, whilst the original co-ordinates when transformed are related to the internal motion. If the collective modes being studied have real physical significance, then it will be found in both these methods that the Hamiltonian is, to a good approximation, separable in the new co-ordinates, and a separation of the collective motion is thereby obtained.

Though these methods are quite successful, they have certain unsatisfactory features. In the Tomonaga method it is generally found that when the Hamiltonian has been separated the problem of finding the eigenvalues of the internal motion part is very difficult. In the superfluous co-ordinate treatment one does not meet with this difficulty but with an equivalent one; this is that it is difficult to find eigenfunctions satisfying the subsidiary conditions. In addition, both theories suffer from the difficulty of not being able to treat very easily the interaction between the collective and internal modes of motion, or the intimately related problem of the damping of the collective motion; where the damping is small this is not a very
DESCRIPTION OF MANY-PARTICLE CORRELATION EFFECTS BY SINGLE-REFERENCE MANY-BODY PERTURBATION THEORY (MBPT)

They key original papers most relevant to this presentation of MBPT are:

1. **The descrip**
   
   **Atomb E**
   
   **Commom**

   **Synopsis**

   The time-independent perturbation theory of quantum mechanics is studied for the case of very large systems, i.e., systems with large spatial dimensions (large volume $\Omega$) and a large number of degrees of freedom. Examples of such systems are met with in the quantum theory of fields, solid state physics, the theory of imperfect gases and in the theory of nuclear matter. Only systems at or near the ground state (i.e., systems at zero temperature) are treated in this paper. In the application of the conventional perturbation theory to such large quantum systems one encounters difficulties which are connected with the fact that even small perturbations produce large changes of the energy and wave function of the whole system. These difficulties manifest themselves through the occurrence of terms containing arbitrarily high powers of the volume $\Omega$ in the perturbation expansion of physical quantities. An extremely bad convergence of the perturbation expansion is the result.

   For the analysis of the $\Omega$-dependence of the terms in the expansion a new formulation of the time-independent perturbation theory is used, which was introduced by N. M. HUGENHOLTZ. Making extensive use of diagrams to represent the different contributions to matrix elements it is possible to locate and separate the $\Omega$-dependent terms, and to carry out partial summations in the original expansion. These separations and summations solve the above difficulties completely. Improved perturbation theoretical expressions are obtained for energies and wave functions of stationary states, as well as for the life-times of metastable states. All terms in these expressions are, in the limit of large $\Omega$, either independent of $\Omega$ or proportional to $\Omega$, corresponding to intensive or extensive physical quantities. The convergence of the improved perturbation expansions is no longer affected by the large magnitude of $\Omega$.

   **Chapter I. Introduction**

   1. **The problem.** This paper is devoted to the perturbation theory of large quantum systems $i.e.$, quantum systems which have large spatial dimensions and a large number of degrees of freedom. The systems met with in the quantum theory of fields are, as is well known, of this type. Also in other branches of physics, such as quantum statistics and the Fermi gas model of heavy nuclei, one has to deal with such large systems. We shall in this paper only be interested in systems at or near the ground state. Our results are, therefore, only applicable to quantum systems at zero temperature.
DESCRIPTION OF MANY-PARTICLE CORRELATION EFFECTS BY SINGLE-REFERENCE MANY-BODY PERTURBATION THEORY (MBPT)

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Hugenholtz, N. M.
1957

"Atomični E

Commun"

Synopsis

The time-independent case of very large D, and a large size in the electron in the theory of state at zero temperature perturbation theory: the energy and wave functions through the D in the perturbation of the perturbation. For the analysis of the timescales above. Making the matrix elements it carry over partial solutions solve the expressions the bases or for the life-time limit of large D, extensive or extensive expansions are no longer.

1. Introduction

In this paper a derivation of the optical-model potential will be presented starting from the Brueckner-Bethe-Goldstone treatment of the nuclear many-body problem. The rigorous equivalence of the optical-model and many-body descriptions of elastic scattering of a nucleon by a nucleus has previously been shown by others. A different approach is adopted here, which results in an explicit prescription for calculating the optical potential by the use of Goldstone diagrams. This prescription involves a linked-cluster (Rayleigh-Schrödinger) expansion similar to that used by Brueckner et al. in calculating properties of static nuclei. The exclusion principle is taken fully into account, and it will be seen that one of its effects on the qualitative nature of the optical potential is immediately evident from the formal expression for the potential. Specifically, the optical potential contains a projection operator which makes the optical wave function orthogonal to the occupied states of the nucleus, as described by a suitably chosen independent-particle model. On the basis of this property of the optical potential it has already been suggested that a modification be made in the usual scattering analysis by means of phenomenological optical-model potentials.

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   For the analysis of the time-independent theory, making an explicit matrix elements carry out partial summations solve the expressions are to solve the life-time limit of large \( O \), extensive or extensive expansions are no longer

   1. The problem large quantum dimensions, and in the quantum density, so different from the collective and the damping of the wave function by the theory to be made is optical-model

   **Abstract**

   A form two-body into scattering of a description terms of a zero of all but one danger exists between state identified from the Hamiltonian perturbation theory for a 1

   **Formulae for Non-degenerate Rayleigh–Schrödinger Perturbation Theory in any order**

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   13th February 1961

   **Abstract**

   It is shown that Bloch's solution for the 4th order perturbation of the energy and the eigenvector in the Rayleigh–Schrödinger perturbation theory of non-degenerate, discrete levels can be expressed in a different form, of a kind suggested by Brueckner, and some advantages of the latter form are presented.

   **§ 1. Introduction**

   The many-body problem has stimulated interest in the systematic formulation of the higher order terms in Rayleigh–Schrödinger perturbation theory for discrete energy levels, i.e. the determination of the discrete eigenstates and eigenvalues of a Hamiltonian:

   \[ H = H_0 + H' \quad \ldots \ldots (1) \]

   (the sum of an unperturbed operator \( H_0 \) and a perturbing one \( H' \)) in the form of series in ascending powers of \( H' \). The case most studied has been that of a system of many particles the interactions between which constitute \( H' \), and important perturbation developments appropriate to this particular case have been made (e.g. Goldstone 1957). However, some attention has also been paid to the formulation of the solution to the general problem (1).

   Bloch (1958) has presented an elegant formulation, which leads to a quite simple expression for the 4th order energy or state vector when the problem is 'non-degenerate' (i.e. when we study the shift of a non-degenerate unperturbed energy level). A different prescription for writing down the energy shifts in the first few perturbation orders (again for the non-degenerate problem) had been suggested by Brueckner (1955), but it was not clear how this was to be generalized correctly to arbitrary order.

   The purpose of this paper is to show that the prescription of Brueckner for the energy can in fact, with small modifications, be extended up to any arbitrary order, and that it can also be adapted to yield formulae for the state vectors to any order. This is achieved by showing that the formulae proved by Bloch can be expressed alternatively in Brueckner's form.

   Brueckner's type of formula has some advantage in the ease with which it can be visualized and applied.

   **§ 2. Bloch's Formulation**

   Let us first summarize the relevant results of Bloch (see also Messiah 1960).

   We consider some unperturbed, discrete eigenvalue of \( H_0 \) say \( E_0 \) which in the first instance may perhaps be degenerate, its eigenvectors spanning a \( d \)-dimensional
DESCRIPTION OF MANY-PARTICLE CORRELATION EFFECTS BY SINGLE-REFERENCE MANY-BODY PERTURBATION THEORY (MBPT)

They key original papers most relevant to this presentation of MBPT are:


KEY THEOREMS OF MBPT

Linked cluster (diagram) theorem (Brueckner, 1955; Goldstone, 1957)

\[
\psi^{(k)} = (R_0 W)^k \phi_0 + \text{renormalization terms} \\
= \left[(R_0 W)^k\right]_{\text{linked}} \phi_0, \quad (k = 1, 2, \ldots), \\
\Delta E^{(k+1)} = \langle \phi_0 | W (R_0 W)^k | \phi_0 \rangle + \text{renormalization terms} \\
= \langle \phi_0 | \left[W (R_0 W)^k\right]_{\text{connected}} | \phi_0 \rangle, \quad (k = 1, 2, \ldots).
\]

Connected cluster theorem (Hubbard, 1957; Hugenholtz, 1957)

\[
\psi = e^T \phi, \quad T = \sum_{k=1}^{\infty} \sum_{C} \{(R_0 W)^k\}_C \\
C \leftrightarrow \text{connected diagrams (including EPV terms)}
\]

- Finite-order MBPT calculations lead to a size extensive description of many-fermion systems, so that no loss of accuracy occurs when the system is made larger.
- One can generate the entire infinite-order MBPT series via the exponential wave function ansatz of coupled-cluster theory, which is size extensive and which can be made size consistent if the reference determinant is separable.
Although the initial proposals suggesting the use of the exponential wave function ansatz of coupled-cluster theory in the context of the many-fermion correlation problem (especially, in nuclei) date back to 1958 and 1960,
the paper that led to the wide-spread use of coupled-cluster theory and its diagrammatic formulation, especially in the context of the many-electron correlation problem, is the 1966 article by J. Čížek:

I. FORMULATION OF THE PROBLEM

Let us consider a system consisting of fixed atomic nuclei and 2n electrons. Let us further exclude from our considerations systems having a degenerate ground state. Then, neglecting relativistic and magnetic effects, the Hamiltonian $H$ of our problem is given by the following equations:

$$\hat{H} = \sum_{i} \epsilon_i \hat{n}_i + \sum_{i<j} v_{ij},$$

$$\sum_{i} \hat{n}_i = N,$$

where $\hat{n}_i$ is a one-particle operator corresponding to the sum of the kinetic and nuclear field energy, and $v_{ij}$ is a two-particle operator of the interelectronic Coulombic repulsion.

Our problem is to find the ground-state eigenvalue and the corresponding eigenfunction of the Hamiltonian

$$\hat{H} | \Phi \rangle = E | \Phi \rangle. \quad (3)$$

In order to solve this problem, the wavefunction $| \Phi \rangle$ is written in the following form

$$| \Phi \rangle = \exp \left( \sum_{i} \hat{T}_i \right) | \Phi_0 \rangle, \quad (4)$$

where $| \Phi \rangle$ is the ground-state wavefunction in the one-particle approximation. This form of the wavefunction is connected with the so-called UHF-type expansion. The use of an expansion of this type for the study of the electronic structure of atoms and molecules was suggested by Sham and coworkers.

In this article, equations for the matrix elements of the operator $\hat{T}_i$ are derived using quantum-field theoretical concepts, namely, (1) the creation and annihilation operators, (2) the hole-particle formulation, (3) the time-independent Wick theorem, and the diagrammatic technique.

If time permits, we will talk about it, at least a little, after discussing MBPT.
THANK YOU


“Algebraic and Diagrammatic Methods for Many-Fermion Systems”

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