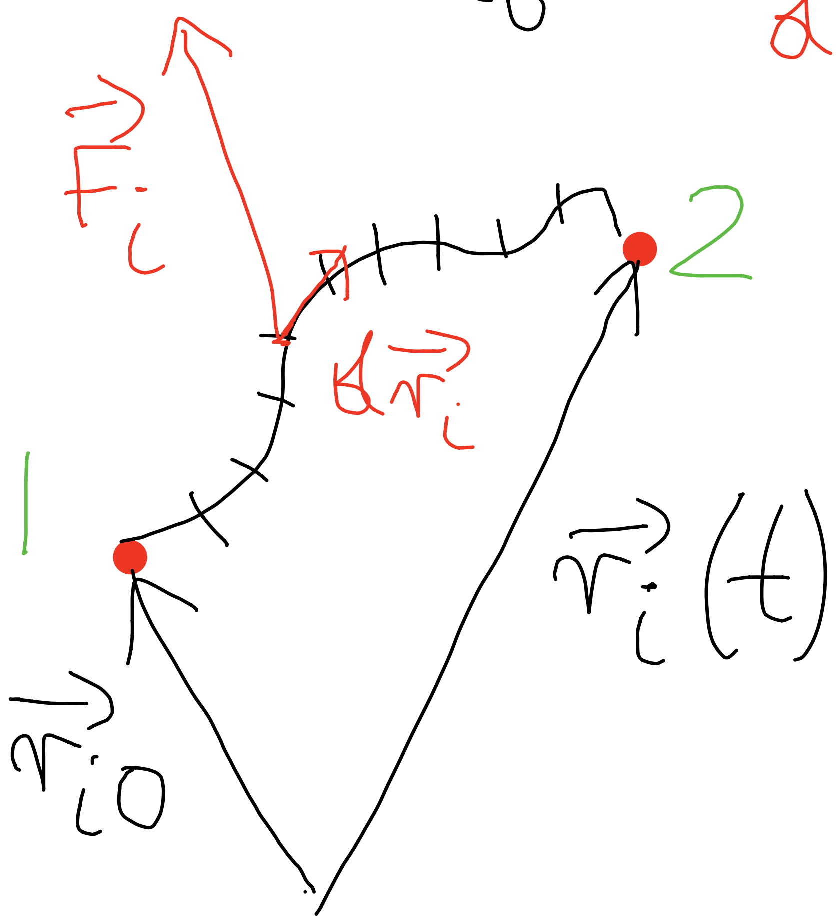
$$V(t) - V(t_0) = -\sum_{i=1}^{N} \int_{t_0}^{t} \overbrace{\vec{F}_i \cdot \underbrace{\dot{\vec{r}}_i \, dt}_{d\vec{r}_i}}^{\vec{F}_i \cdot d\vec{r}_i}$$



$$\overset{V_2}{\overset{\shortparallel}{V(\vec{r}_1, \dots, \vec{r}_N)}} - \overset{V_1}{\overset{\shortparallel}{V(\vec{r}_{10}, \dots, \vec{r}_{N0})}}$$

$$= - \sum_{i=1}^{N} \int_{\vec{r}_{i0}}^{\vec{r}_i} \vec{F}_i \cdot d\vec{r}_i$$

$N=1$, 1D

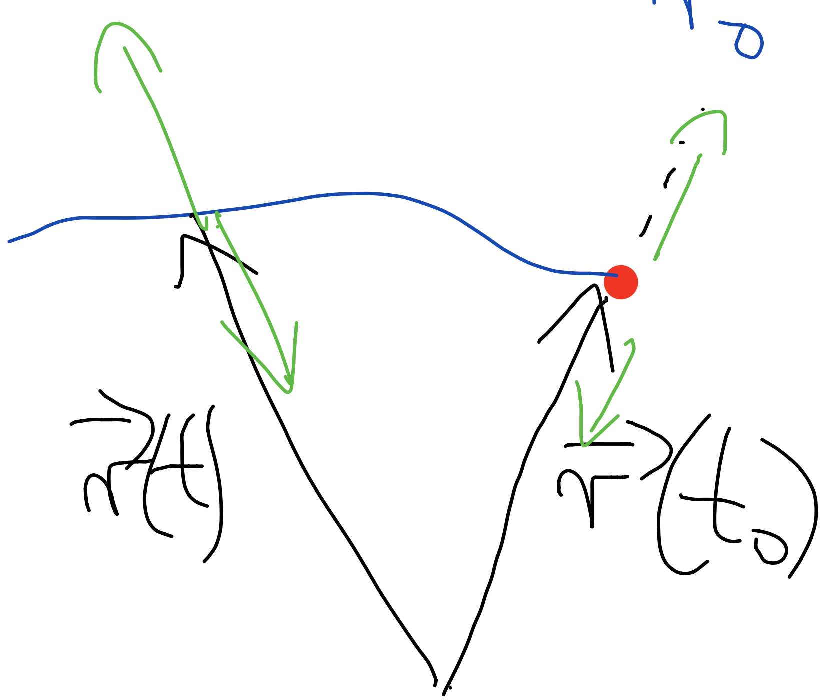
$$F_x = m\ddot{x}$$

$$V(x) - V(x_0) =$$

$$= - \int_{x_0}^{x} F_{\tilde{x}}\, d\tilde{x}$$

$N=1$, 3D

$$V(\vec{r}) - V(\vec{r}_0) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{r}$$



$$\vec{F} = F_r \frac{\vec{r}}{r}$$

$$V(\vec{r}) - V(\vec{r}_0) \qquad r\,dr$$

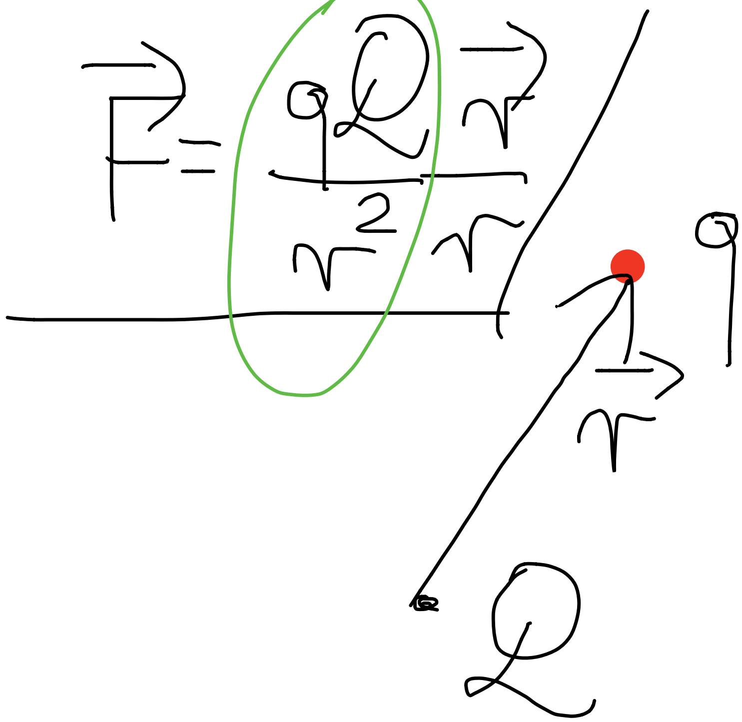
$$= -\int_{\vec{r}_0}^{\vec{r}} F_r \frac{\vec{r}}{r} \cdot d\vec{r}$$

$$\vec{r} \cdot d\vec{r} = \frac{1}{2} d(\vec{r} \cdot \vec{r})$$

$$= \frac{1}{2} d(r^2) = r\,dr$$

$$V(r) - V(r_0) = -\int_{r_0}^{r} F_r \, dr$$

$$\vec{F} = \frac{qQ}{r^2}\frac{\vec{r}}{r}$$



$$F_r = \frac{qQ}{r^2}$$

$$V(r) - V(r_0) = -\int_{r_0}^{r} \frac{qQ}{r^2}\,dr$$

$$V(r) = V(r_0) - \int_{r_0}^{r} \frac{qQ}{r^2}\,dr$$

$$V(r) = V(r_0) + \left.\frac{qQ}{r}\right|_{r_0}^{r}$$

$$V(r) = \frac{qQ}{r} + \text{const.}$$

$$\lim_{r \to \infty} V(r) = 0$$

$$V(r) = \frac{qQ}{r}$$

$$\vec{F}_i^{\,tot} = \vec{F}_i + \vec{F}_i^{\,c}$$

$$i = 1, \dots, N$$

$$X_j^{tot} = X_j + X_j^c \qquad j = 1, \dots, 3N$$

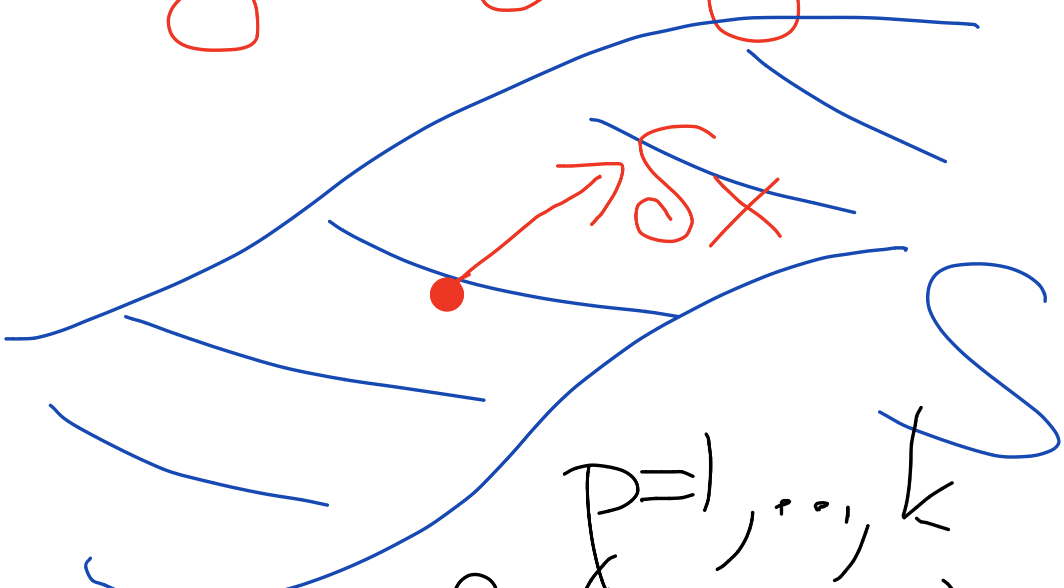
$$f_p(x_1, \dots, x_{3N}, t) = 0 \qquad p = 1, \dots, k$$

f dimensional

$$X^c \perp \delta x$$

$$X^c \cdot \delta x = 0$$

$$\sum_{j=1}^{3N} X^c_j \, \delta x_j = 0$$



$$p = 1, \ldots, k$$

$$f_p(x_1, \ldots, x_{3N}, t) = 0$$

$$\nabla f_p \perp S$$

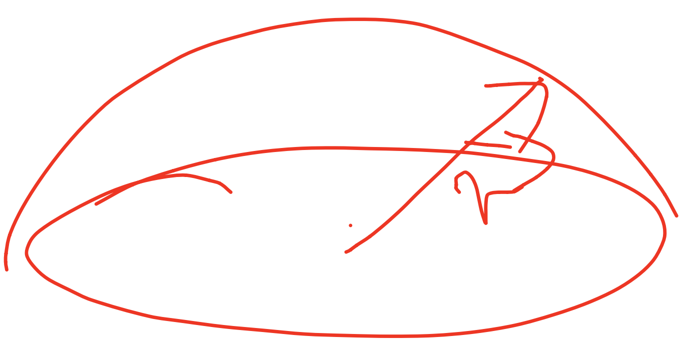
$$\boxed{\nabla f_p = \left(\frac{\partial f_p}{\partial x_1}, \ldots, \frac{\partial f_p}{\partial x_{3N}}\right)}$$

$$f(x,y,z) = x^2 + y^2 + z^2 - a^2$$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

$$= (2x, 2y, 2z)$$

$$= 2\vec{r}$$



$$\nabla f_p \perp \delta x$$

$$p = 1, \ldots, N$$

$$\nabla f_p \cdot \delta x = 0$$

$$\boxed{\sum_{j=1}^{3N} \frac{\partial f_p}{\partial x_j} \delta x_j = 0}$$

$$-X_j^c = X_j - m_j\ddot{x}_j$$

$$\sum_j \delta x_j$$

$$0 = -\sum_{j=1}^{3N} X_j^c\,\delta x_j$$

$$= \sum_{j=1}^{3N}\left(X_j - m_j\ddot{x}_j\right)\delta x_j$$

$$\boxed{\sum_{j=1}^{3N}\left(X_j - m_j\ddot{x}_j\right)\delta x_j = 0}$$