Photons:

$$E = mc^2$$

$$m = m_0 \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}$$

$$\vec{p} = m\vec{v}, \quad p = mv$$

$$p = \frac{E}{c^2} v$$

$$E = h\nu, \quad p = \frac{h\nu}{c^2} v$$

$$v = c$$

$$p = \frac{h\nu}{c^{\cancel{2}}} \cancel{c} = \frac{h\nu}{c}$$

$$\lambda\nu = c$$

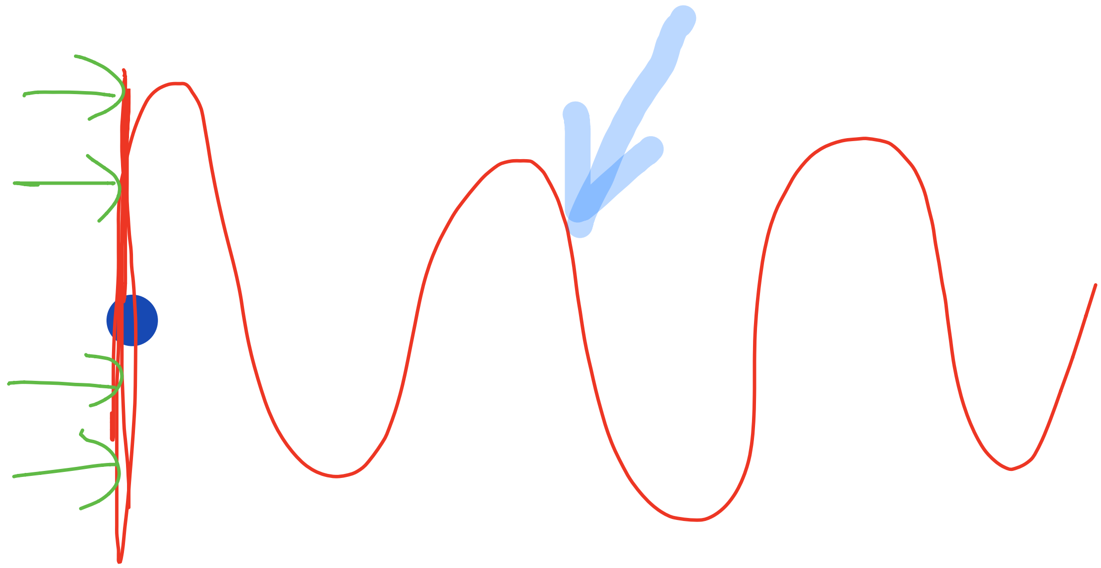
$$p = \frac{h}{\lambda}$$

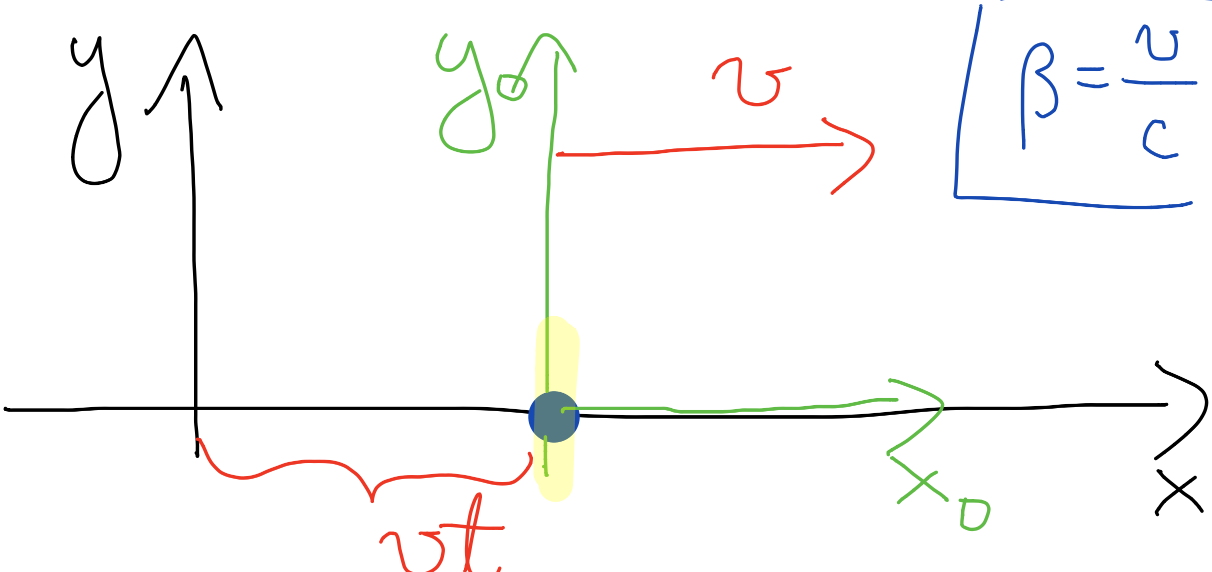
Particles (with a mass)
(de Broglie) ~1923

$$E = mc^2$$

$$m = m_0 \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}$$

$$E_0 = m_0 c^2$$ (rest energy)





$$\boxed{\beta = \frac{v}{c}}$$

$$x_0 = \frac{x - vt}{\sqrt{1-\beta^2}}, \quad y_0 = y$$

$$z_0 = z \qquad t_0 = \frac{t - \frac{v}{c^2}x}{\sqrt{1-\beta^2}}$$

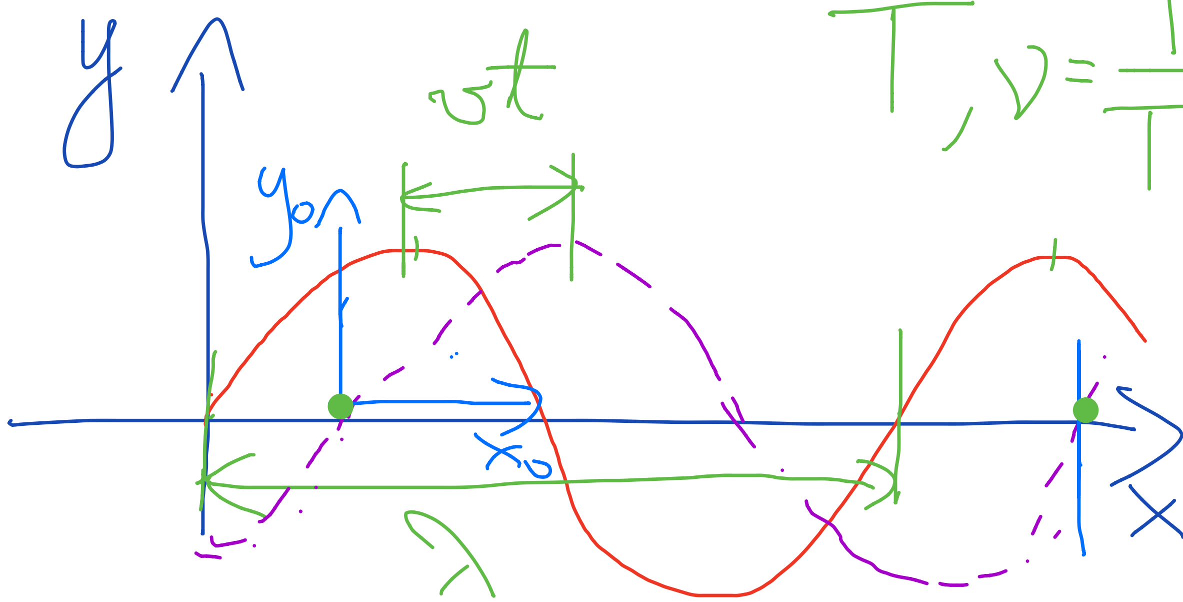
$$\psi = A \sin(2\pi\nu_0)t_0$$

$$A = A(x_0, y_0, z_0)$$

$$\psi(x,y,z,t) =$$

$$= A\left(\frac{x - vt}{\sqrt{1-\beta^2}}, y, z\right) \times$$

$$\times \sin 2\pi\nu_0 \left(\frac{t - \frac{v}{c^2}x}{\sqrt{1-\beta^2}}\right)$$



$T, \nu = \frac{1}{T}$

$$v = \frac{\lambda}{T} = \lambda\nu$$

$$x_0 = x - vt$$

$$A \sin \frac{2\pi}{\lambda} x_0 =$$

$$= A \sin\left(\frac{2\pi}{\lambda} x - \overbrace{\frac{2\pi}{\lambda} v}^{2\pi\nu} t\right)$$

$$\frac{2\pi}{\lambda} = k \qquad 2\pi\nu = \omega$$

$$A\sin(kx - \omega t)$$

$$\cancel{2\pi}\frac{1}{\lambda}\cancel{x} = \frac{\cancel{2\pi}\nu_0}{c^2}\frac{v\cancel{x}}{\sqrt{1-\beta^2}}$$

$$\frac{1}{\lambda} = \frac{\nu_0}{c^2}\frac{v}{\sqrt{1-\beta^2}}$$

$$\cancel{2\pi}\nu_0\frac{\cancel{t}}{\sqrt{1-\beta^2}} = \cancel{2\pi}\nu\cancel{t}$$

$$\nu = \frac{\nu_0}{\sqrt{1-\beta^2}}$$

$$E = mc^2 = m_0 c^2 \frac{1}{\sqrt{1-\beta^2}}$$

$$= E_0 \frac{1}{\sqrt{1-\beta^2}}$$

$$\frac{E}{\nu} = \frac{E_0}{\cancel{\sqrt{1-\beta^2}}} \frac{\cancel{\sqrt{1-\beta^2}}}{\nu_0} = \frac{E_0}{\nu_0}$$

$$= \text{const.} \; (= h) \, !$$

$$E = h\nu, \; E_0 = \underline{\underline{h\nu_0}}$$

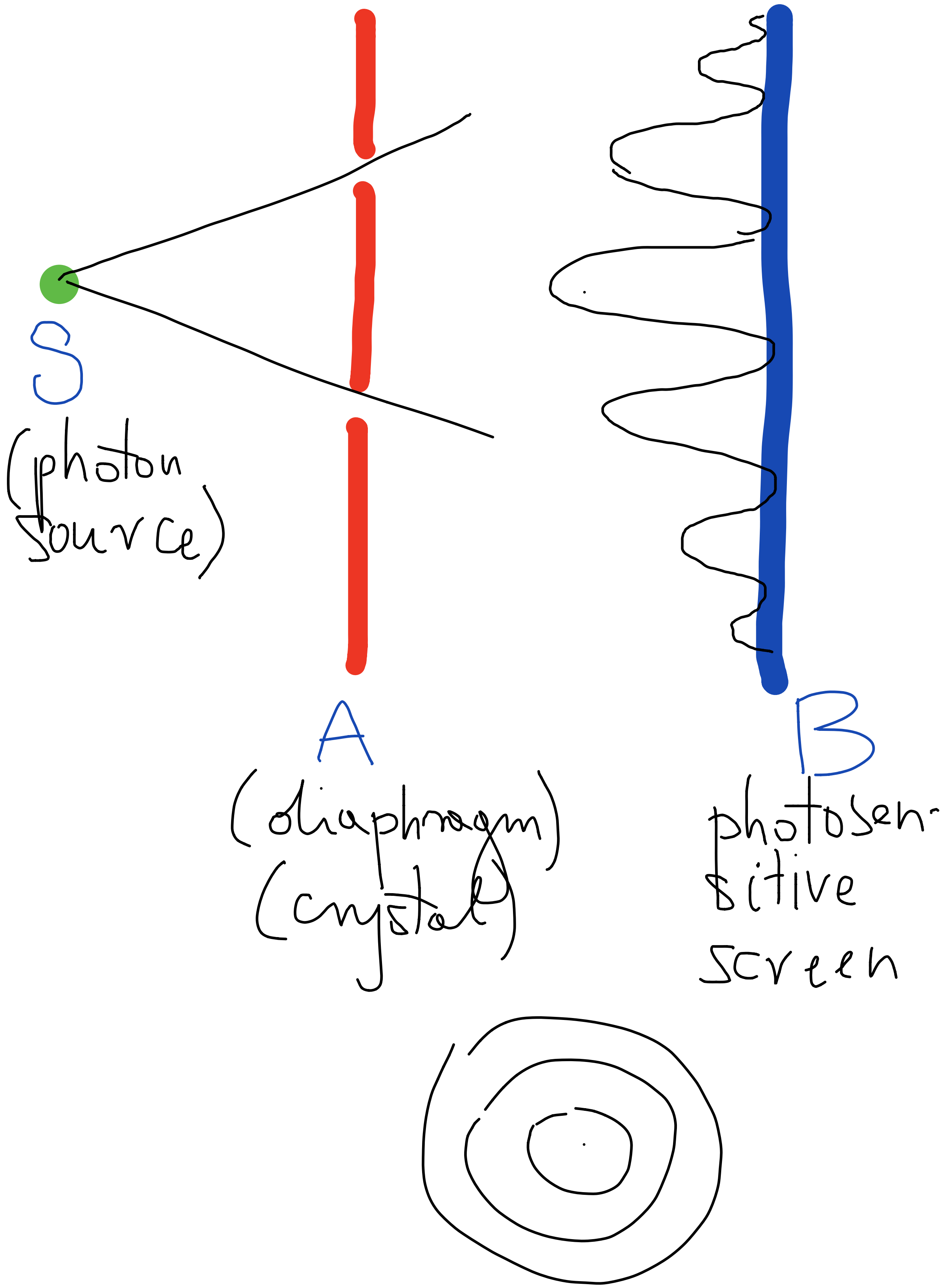
$$\frac{1}{\lambda} = \frac{\nu_0}{c^2} \frac{v}{\sqrt{1-\beta^2}} \Big/ h$$

$$\frac{h}{\lambda} = \underbrace{\frac{h\nu_0}{m_0 c^2}}_{=1} \frac{m_0 v}{\sqrt{1-\beta^2}}$$

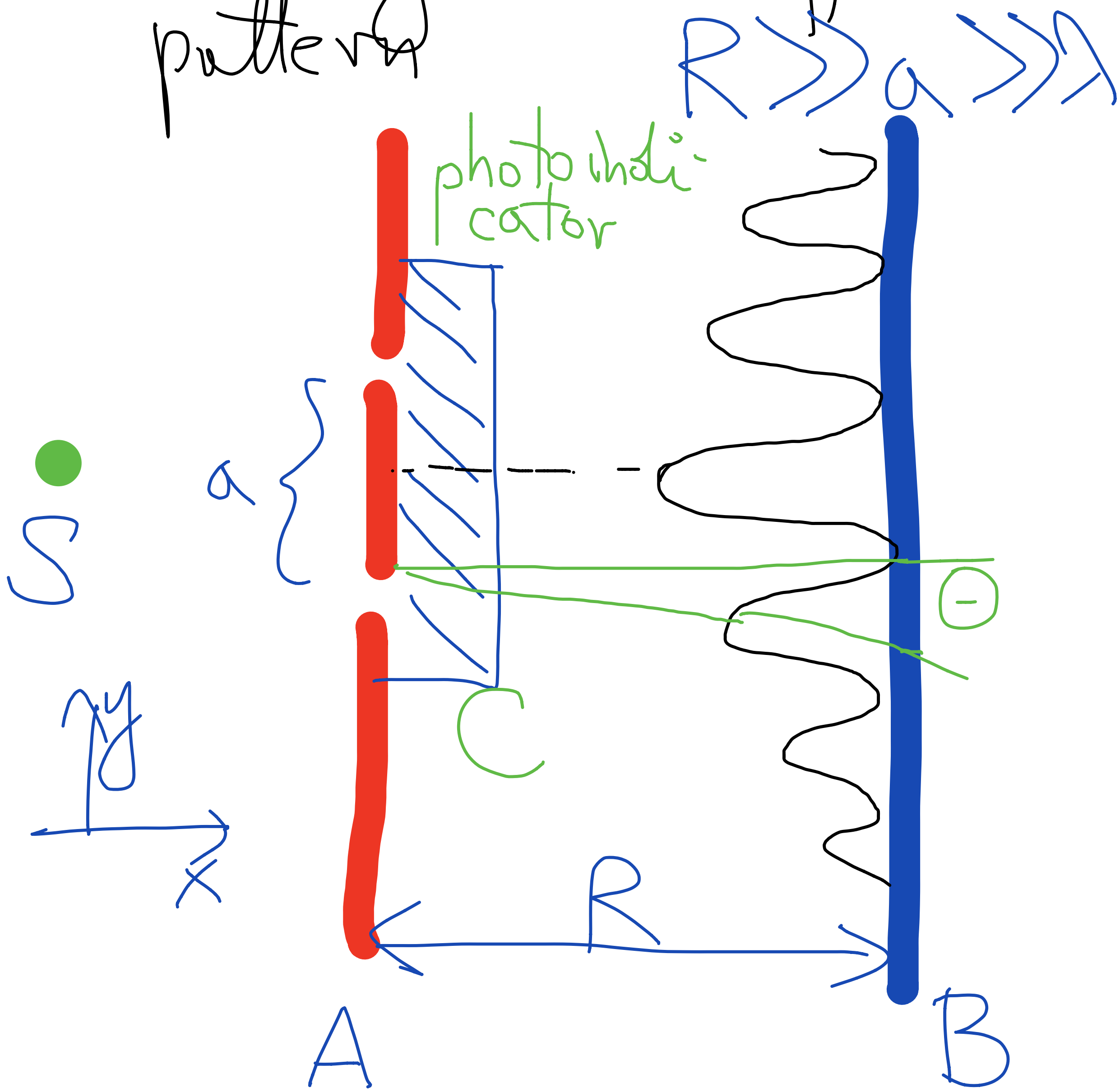
$$\frac{h}{\lambda} = \frac{m_0 v}{\sqrt{1-\beta^2}} = mv$$

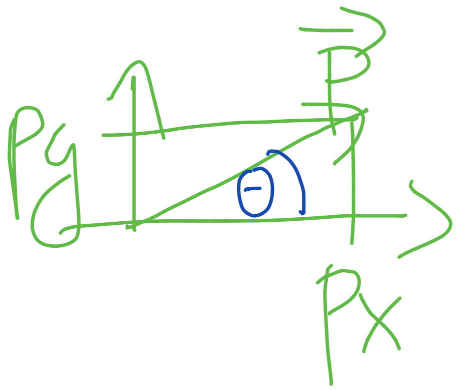
$$= p$$

$$E = h\nu, \quad p = \frac{h}{\lambda} \,!$$


S
(photon source)
A
(diaphragm)
(crystal)
B
photosen-
sitive
screen

An attempt to locate a particle (photon) destroys the diffraction pattern





$$\frac{p_y}{p} = \sin\theta$$

$$p_y = p\sin\theta \approx p\theta$$

If we don't want to destroy the diffraction pattern by using C

$$\Delta p_y \ll p_y$$

$$\Delta p_y \ll \theta p$$

$$\theta = \frac{\lambda}{2a}$$

$$\Delta p_y \ll \frac{\lambda}{2a} p \checkmark$$

To locate the photon and determine which slit the photon goes through

$$\Delta y < \frac{1}{2} a$$

$$\Delta y \, \Delta p_y \ll \frac{1}{2} \cancel{a} \, \frac{\lambda}{2 \cancel{a}} p$$

$$= \frac{\lambda}{4} p = \frac{\cancel{\lambda}}{4} \frac{h}{\cancel{\lambda}}$$

$$= \frac{h}{4}$$

$\Delta y \Delta p_y << \frac{1}{4} h$

contradicts the uncertainty principle

Determination of the particle position destroys the diffraction pattern.

Wave packets
(wave functions)

$\psi = \psi(x, y, z, t)$
for a single particle

See conditions
1, 2, 3 on p. 110 in
the lecture notes

$$\psi(x,t) = \sum_{\omega,k} A_{k,\omega} e^{i(kx-\omega t)}$$

$$(\sin(kx-\omega t))$$

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$

Example: superposition of 2 waves

$$\psi_1 = A e^{i(k_1 x - \omega_1 t)}$$

$$\psi_2 = A e^{i(k_2 x - \omega_2 t)}$$

$$\psi_1 + \psi_2 = A \times$$

$$\times\, e^{\frac{1}{2} i [(k_1+k_2)x - (\omega_1+\omega_2)t]}$$

$$\times \left\{ e^{\frac{1}{2} i [(k_1-k_2)x - (\omega_1-\omega_2)t]} + e^{\frac{1}{2} i [(k_2-k_1)x - (\omega_2-\omega_1)t]} \right\}$$

$$= 2A\, e^{\frac{1}{2} i [(k_1+k_2)x - (\omega_1+\omega_2)t]}$$

$$\times \cos\left[\frac{k_1-k_2}{2} x - \frac{\omega_1-\omega_2}{2} t\right]$$

$$e^{i\alpha} + e^{-i\alpha} = 2\cos\alpha \checkmark$$

$$\psi_1 + \psi_2 = A_{group} e^{i(\bar{k}x - \bar{\omega}t)}$$

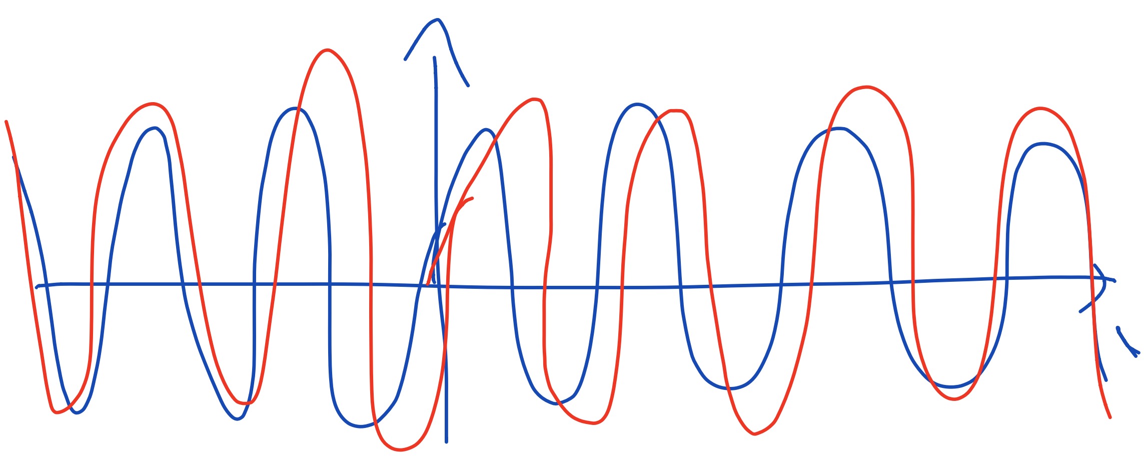
$$A_{group} = 2A \cos\left[\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right]$$

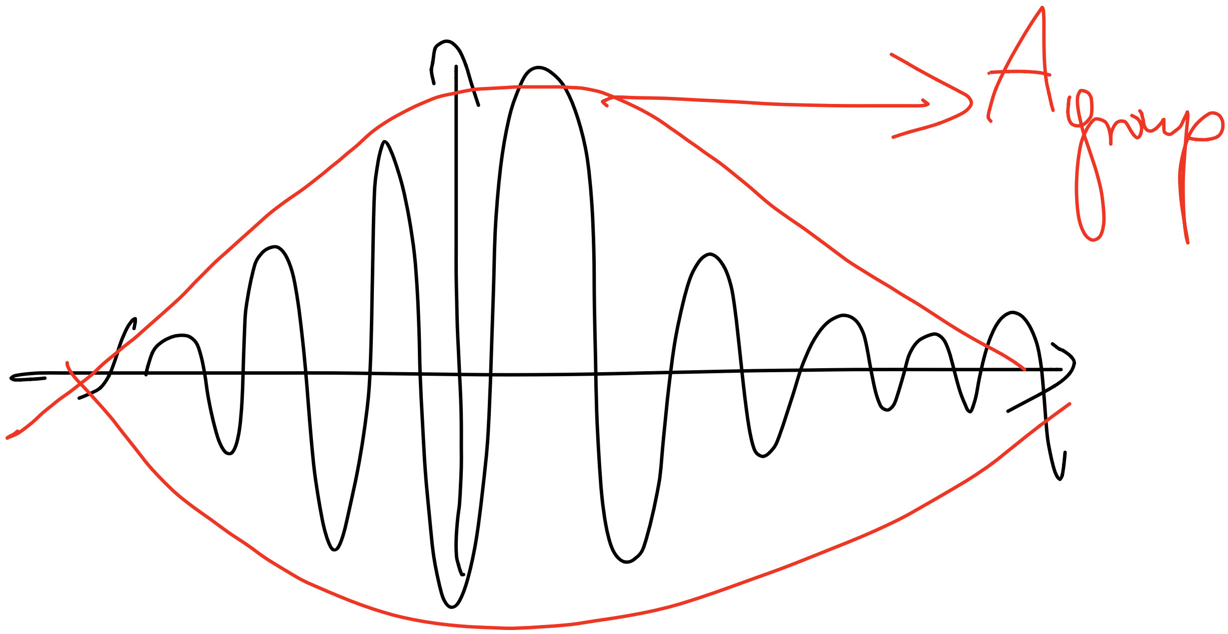
$$\Delta k = k_1 - k_2, \quad \Delta\omega = \omega_1 - \omega_2$$

$$\bar{k} = \frac{k_1 + k_2}{2}, \quad \bar{\omega} = \frac{\omega_1 + \omega_2}{2}$$

$$k_1 \approx k_2, \quad \omega_1 \approx \omega_2$$

$\Delta k, \Delta\omega$ - small





$$v = \frac{\lambda}{T} = \lambda\nu = \frac{\lambda}{2\pi} 2\pi\nu$$

$$= \frac{\omega}{k}$$

$$v_{group} = \frac{\frac{\Delta\omega}{2}}{\frac{\Delta k}{2}} = \frac{\Delta\omega}{\Delta k}$$

$$\boxed{v_{group} = \frac{d\omega}{dk}}$$

$$v_{group} = \frac{\hbar\, d\omega}{\hbar\, dk}, \quad \hbar = \frac{h}{2\pi}$$

$$= \frac{d(\hbar\omega)}{d(\hbar k)}$$

$$\hbar\omega = \frac{h}{\cancel{2\pi}}\cancel{2\pi}\nu = h\nu = E$$

$$= \frac{p^2}{2m}$$

$$\hbar k = \frac{h}{\cancel{2\pi}}\frac{\cancel{2\pi}}{\lambda} = \frac{h}{\lambda} = p$$

$$v_{group} = \frac{dE}{dp} = \frac{d\left(\frac{p^2}{2m}\right)}{dp}$$

$$= \frac{p}{m} = v_{classical}$$

$$\boxed{\begin{array}{l}\underline{\text{Next time:}} \\ \text{Consistency of the wave-packet description} \\ \text{with the uncertainty principle}\end{array}}$$