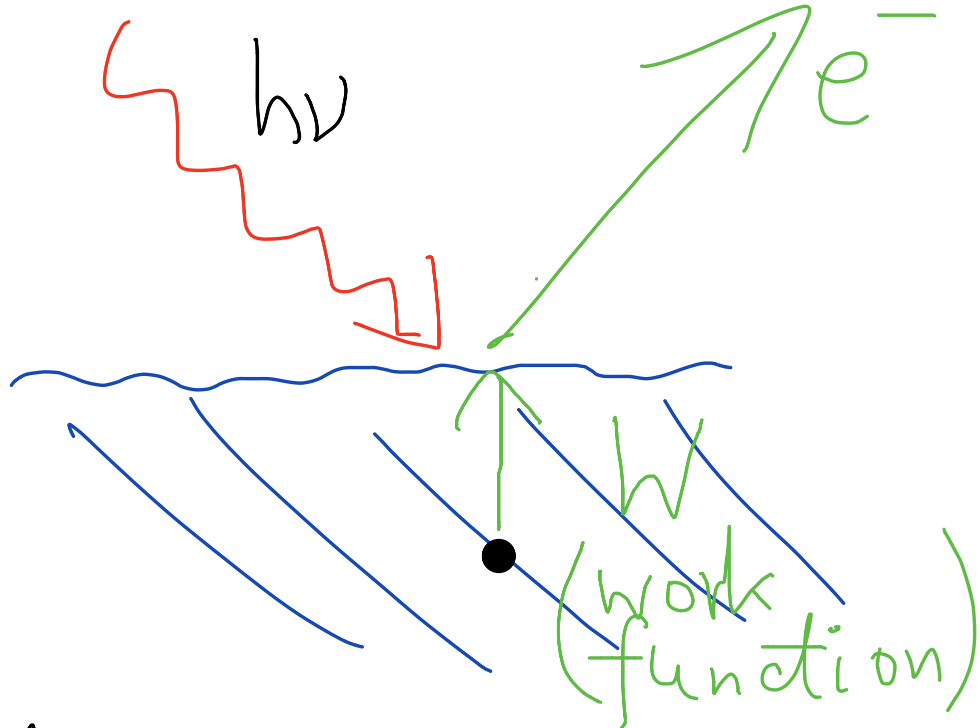
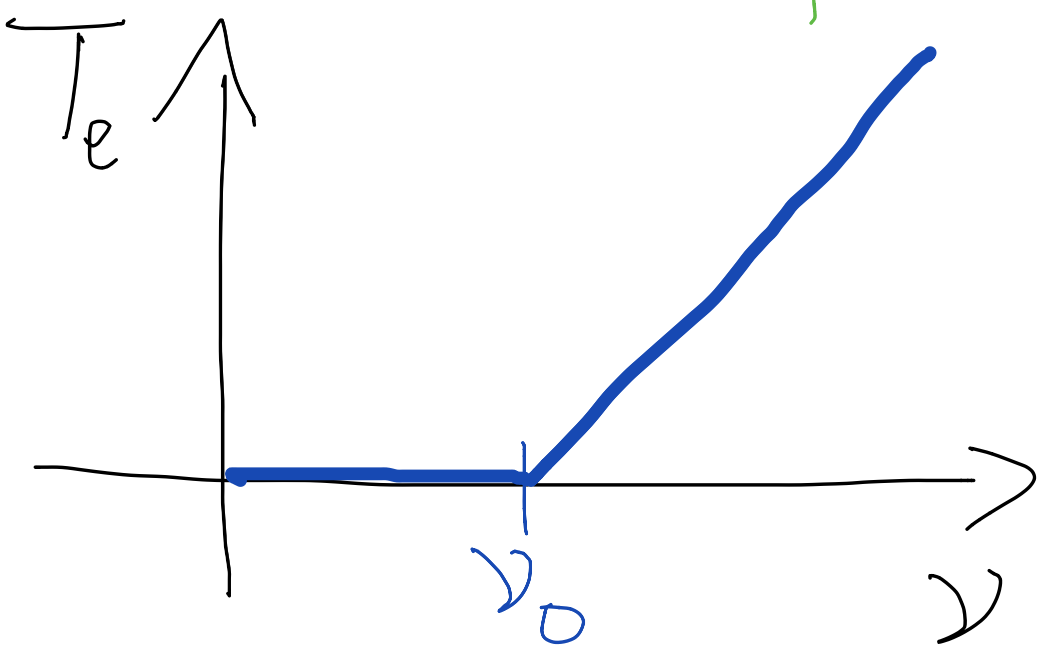
Photoelectric effect





$$\nu \geq \nu_0$$

$$h\nu = W + \frac{m_e v^2}{2}$$

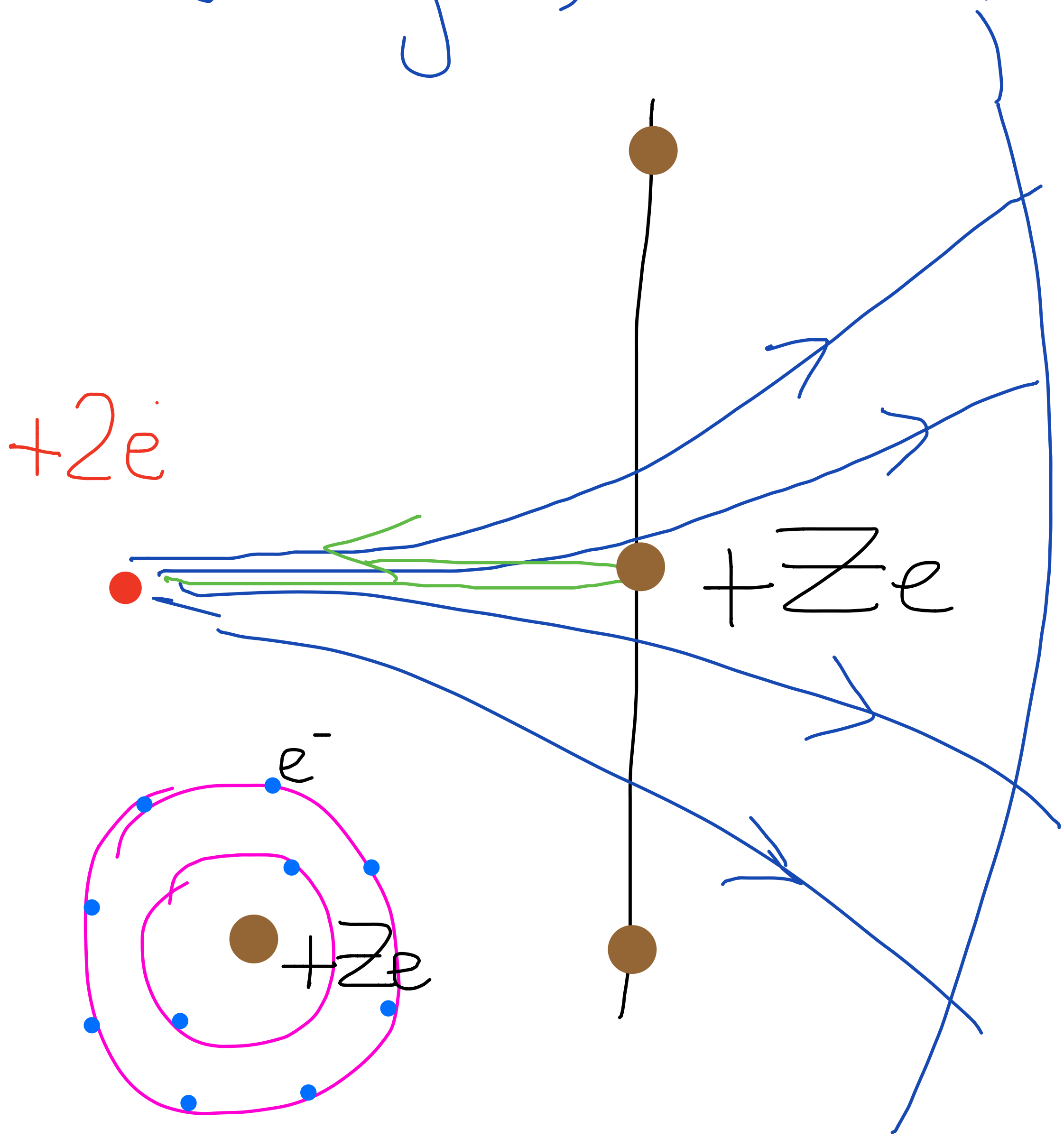
$$= W + T_e$$

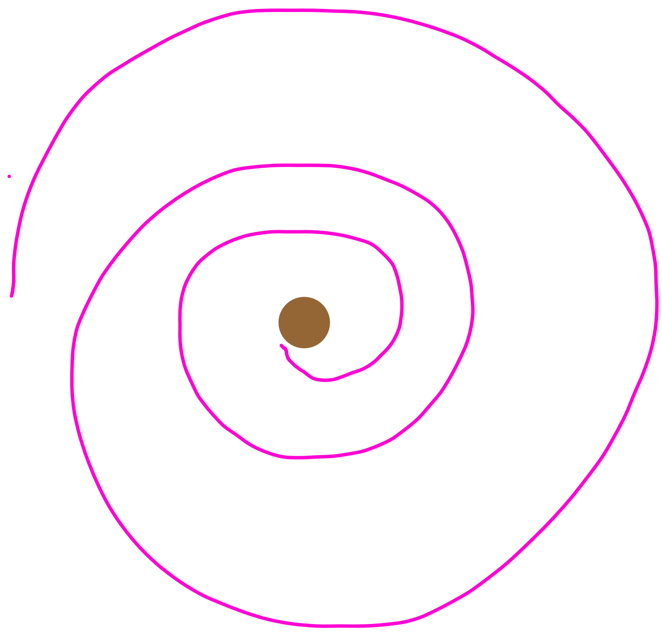
$$W = h\nu_0$$

$$h\nu = h\nu_0 + T_e$$

$$h(\nu - \nu_0) = T_e$$

Rutherford
(Geiger) – 1911

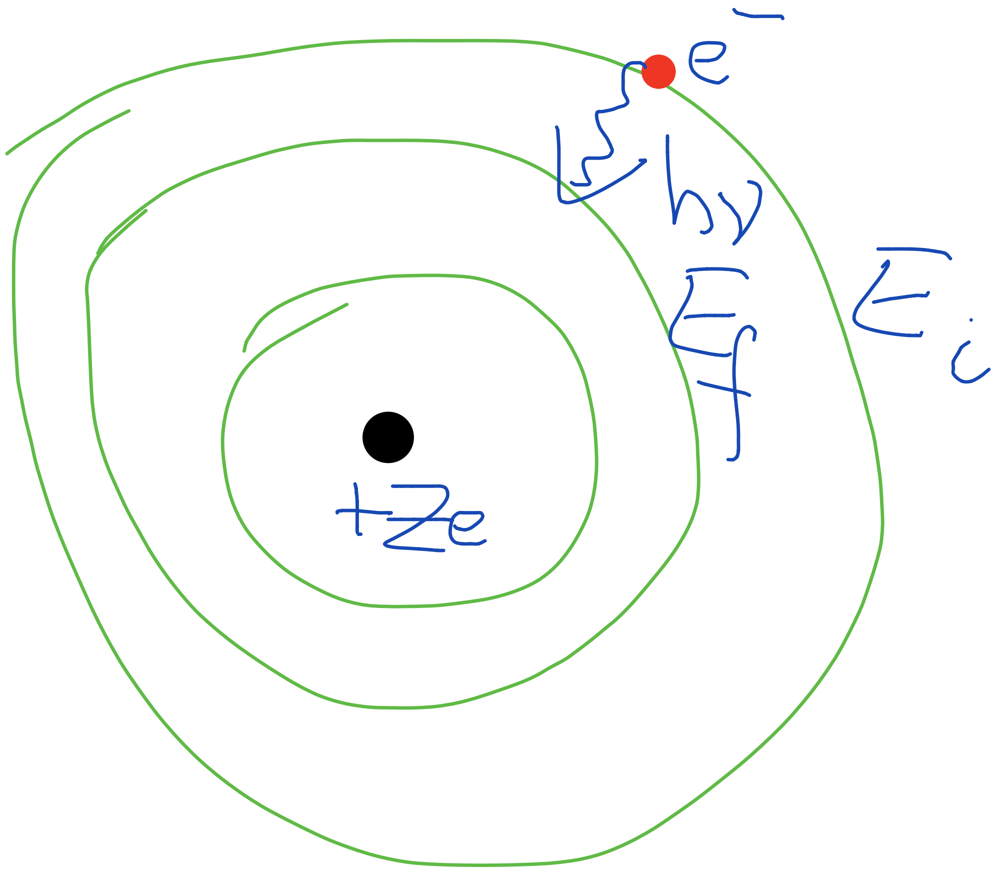




Bohr
1913

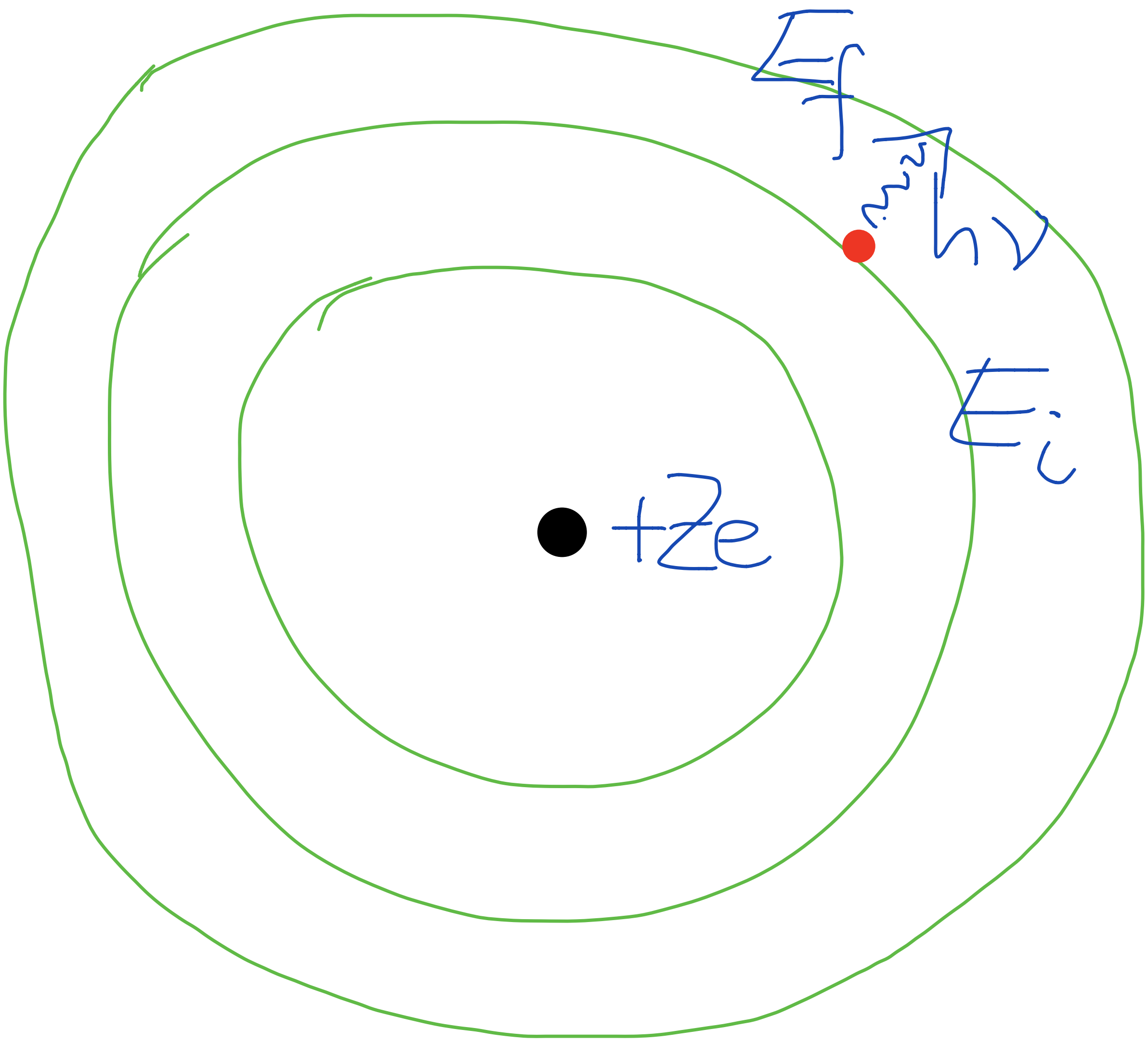
1. Existence of stationary states (quantum states).

2. Bohr's frequency condition.



$$h\nu = E_i - E_f$$

(emission)



$$E_f - E_i = h\nu$$

(absorption)

Old quantum theory

Bohr, Planck,
Wilson, Sommerfeld

Hamilton-Jacobi

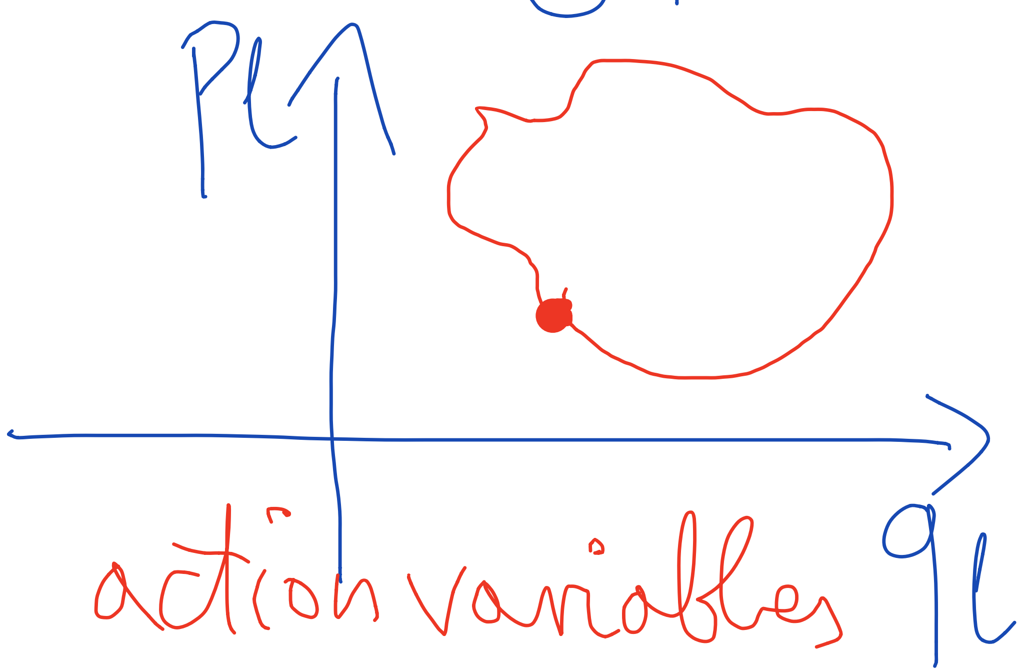
✓ separable
✓ multi-periodic systems

$$q_1, \ldots, q_f$$

$$p_1, \ldots, p_f$$

$$p_\ell = p_\ell(q_\ell)$$

$$J_\ell = \oint p_\ell(q_\ell)\, dq_\ell$$



integer

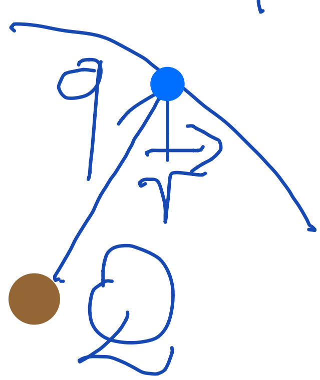
$$J_\ell = n_\ell h$$

$$\ell = 1, \dots, f$$

$$q = -e, \quad Q = Ze$$

$$\alpha = qQ < 0$$

$$\vec{F} = F_r \frac{\vec{r}}{r}$$



$$F_r = \frac{\alpha}{r^2}$$

$\alpha < 0$ attraction

$\alpha > 0$ repulsion

$$\left(F_r = -G \frac{mM}{r^2} \right)$$

$$\alpha = -GmM$$

Kepler problem

$$V = \frac{\alpha}{r}$$

V $\alpha < 0$

$E < 0$

$\cancel{E = 0}$

$E > 0$

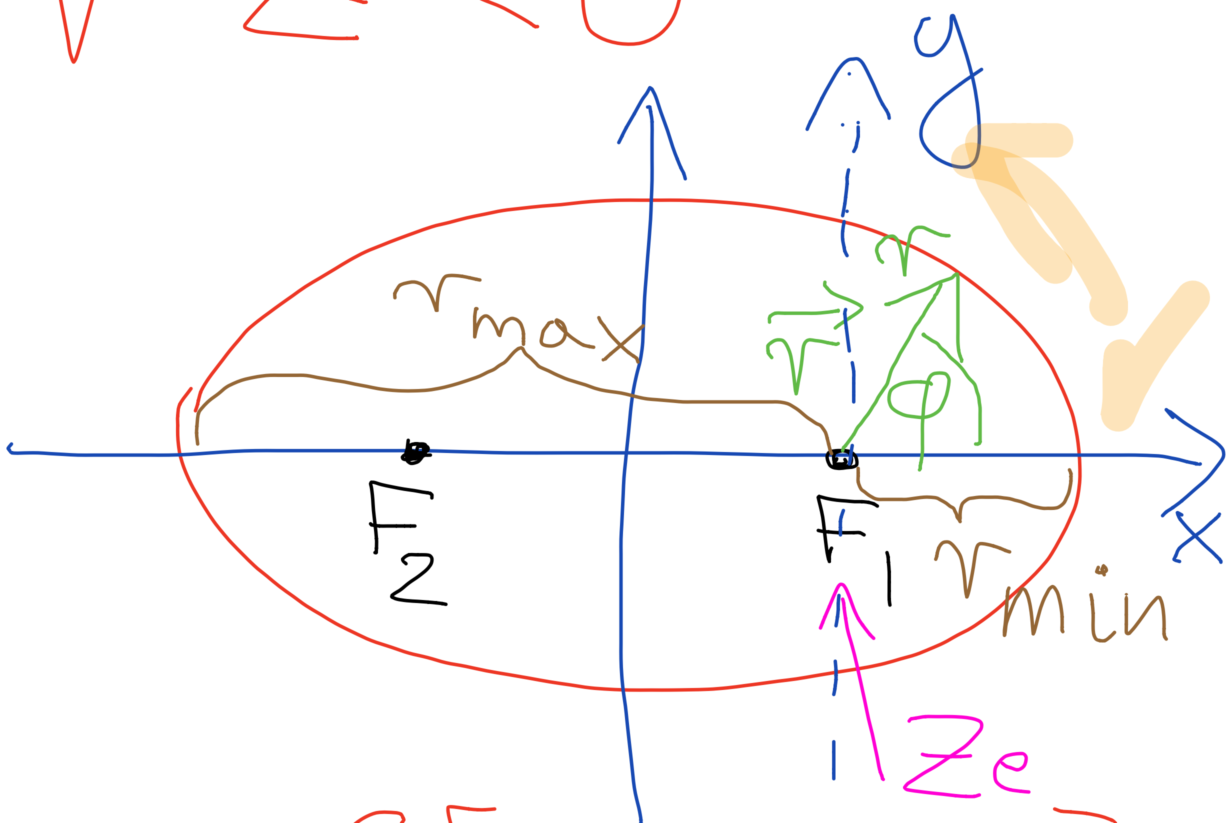
V $\alpha > 0$

$E = T + V > 0$

(scattering)

$\alpha < 0$ (attraction)

$\surd$ $E < 0$



$r \in [r_{min}, r_{max}]$

$\varphi \in [0, 2\pi)$

(bound states)

$\alpha < 0$

$E > 0$

scattering

$$\frac{d\vec{L}}{dt} = \vec{D} = \vec{r} \times \vec{F}$$

$$= 0$$

$$\left(\vec{r} \times \vec{F} = = \frac{1}{r} F_r (\vec{r} \times \vec{r}) = 0 \right)$$

$$\vec{L} = \text{const.}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$= m(\vec{r} \times \dot{\vec{r}})$$

$$= \text{const.} \quad \boxed{\text{planar motion}}$$

$f = 2$, polar coordinates

$$x = r\cos\phi$$

$$y = r\sin\phi$$

$$T = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\varphi}^2\right)$$

$$L = T - V$$

$$= \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\varphi}^2\right) - \frac{\alpha}{r}$$

$$\checkmark \quad p_r = \frac{\partial L}{\partial \dot{r}}$$

$$= m\dot{r}$$

$$\checkmark \quad p_\varphi = \frac{\partial L}{\partial \dot{\varphi}} = mr^2\dot{\varphi}$$

$L_z \to$

$$\begin{cases} \dot{p}_r = \dfrac{\partial L}{\partial r} \\ \qquad = m r \dot{\varphi}^2 - \dfrac{dV}{dr} \\ \dot{p}_\varphi = \dfrac{\partial L}{\partial \varphi} = 0 \end{cases}$$

Lagrange eqs.

$$\boxed{\frac{d}{dt} \underbrace{\frac{\partial L}{\partial \dot{q}_l}}_{= p_l} = \frac{\partial L}{\partial q_l}}$$

$$p_\varphi = \text{const.} = J$$

$$E = T + V$$

$$= \frac{1}{2m}\left(p_r^2 + \frac{p_\varphi^2}{r^2}\right)$$

$$+ V(r) = \text{const.}$$

$$V = \frac{\alpha}{r}$$

$$J_r = \oint p_r \, dr$$

$$= 2 \int_{r_{min}}^{r_{max}} p_r \, dr$$

$$J_\varphi = \oint p_\varphi \, d\varphi$$

$$= \int_0^{2\pi} p_\varphi \, d\varphi$$

$$= 2\pi J$$

$$E = \frac{1}{2m}\left(p_r^2 + \frac{p_\varphi^2}{r^2}\right) + \frac{\alpha}{r}$$

J

$$p_r = \sqrt{2m\left(E - \frac{\alpha}{r}\right) - \frac{J^2}{r^2}}$$

$$p_r = p_r(r)$$

$$p_\varphi = \text{const.} \ (= J)$$

$$J_r = 2\int_{r_{min}}^{r_{max}} \sqrt{2m\left(E - \frac{\alpha}{r}\right) - \frac{J^2}{r^2}}\, dr$$

$$J_\varphi = 2\pi J$$

$$J_r = -2\pi J + \frac{2\pi m\alpha}{\sqrt{-2mE}}$$

$$(E<0)$$

$$J_\varphi = 2\pi J$$

$$\begin{cases} J_r = -J_\varphi + \dfrac{2\pi m \alpha}{\sqrt{-2mE}} \\ J_\varphi = 2\pi J \end{cases}$$

$$E = -\frac{\overset{2}{\cancel{4}}\pi^2 m^{\cancel{2}} \alpha^2}{\cancel{2}\cancel{m}(J_r + J_\varphi)^2}$$

$$\begin{cases} E = -\dfrac{2\pi^2 m\alpha^2}{(J_r + J_\varphi)^2} \\ J = \dfrac{J_\varphi}{2\pi} \end{cases}$$

E, J in terms of J_r, J_φ

(as anticipated)

$$J_\ell = n_\ell h$$

$$J_r = n_r h$$

$$J_\varphi = n_\varphi h$$

$$n_r, n_\varphi = 0, \pm 1, \pm 2, \ldots$$

$$J_r = \oint p_r \, dr$$

$$= \oint m \dot{r} \frac{dr}{dt} dt$$

$\dot{r}$

$$J_r = \oint m(\dot{r})^2 dt$$

$$> 0$$

$$n_r = 0, 1, 2, \ldots$$

$$J_\varphi = \oint p_\varphi d\varphi$$

$$= \oint m r^2 \dot{\varphi} \frac{d\varphi}{dt} dt$$

$$\dot{\varphi}$$

$$\parallel$$

$$J_\varphi = \oint m r^2 (\dot{\varphi})^2 dt$$

$$> 0$$

$$n_\varphi = \cancel{0}, 1, 2, \ldots$$

Can n_φ be 0?

$$\dot{\varphi} = 0 \implies$$

$$\implies \varphi = \text{const.}$$

NO

$$n_r = 0, 1, 2, \ldots$$

$$n_\varphi = 1, 2, \ldots$$

$$E = -\frac{2\pi^2 m\alpha^2}{(J_r + J_\varphi)^2}$$

$$J_r = n_r h, \quad J_\varphi = n_\varphi h$$

$$n_r = 0, 1, 2, \ldots$$

$$n_\varphi = 1, 2, \ldots$$

$$J = \frac{J_\varphi}{2\pi}$$

$$E = -\frac{2\pi^2 m \alpha^2}{(n_r + n_\varphi)^2 h^2}$$

$$J = n_\varphi \frac{h}{2\pi} = n_\varphi \hbar$$

$$n_r + \overset{k}{n_\varphi} = n$$

principal quantum number

$n = 1, 2, \ldots$

$k \equiv n_\varphi = 1, 2, \ldots$

azimuthal quantum number

$$E_n = \frac{-2\pi^2 m \alpha^2}{n^2 h^2}$$

$$= -\frac{m\alpha^2}{2n^2\hbar^2}$$

$$\hbar = \frac{h}{2\pi}$$

$$J = k\hbar$$

$$\alpha = qQ = -e \times Ze$$
$$= -Ze^2$$

$$E_n = -\frac{m_e Z^2 e^4}{2n^2\hbar^2}$$

$$J = k\hbar$$

$$n = 1, 2, \ldots \quad (n_r \geq 0)$$

$$k = 1, \ldots, n$$

$\boxed{\text{QM}}$

$$L^2 = l(l+1)\hbar^2$$

$$l = 0, 1, \ldots, n-1$$

$$L = \sqrt{l(l+1)}\,\hbar$$

$$L_z = m\hbar$$

$$m = -l, \ldots, l$$

We used r, φ

How about using

r, ϑ, φ?

$$L = k\hbar$$

$$k = 1, \ldots, n$$

$$L_z = m\hbar$$

$$m \in (-k, k)$$

(see pages 99 and 100)