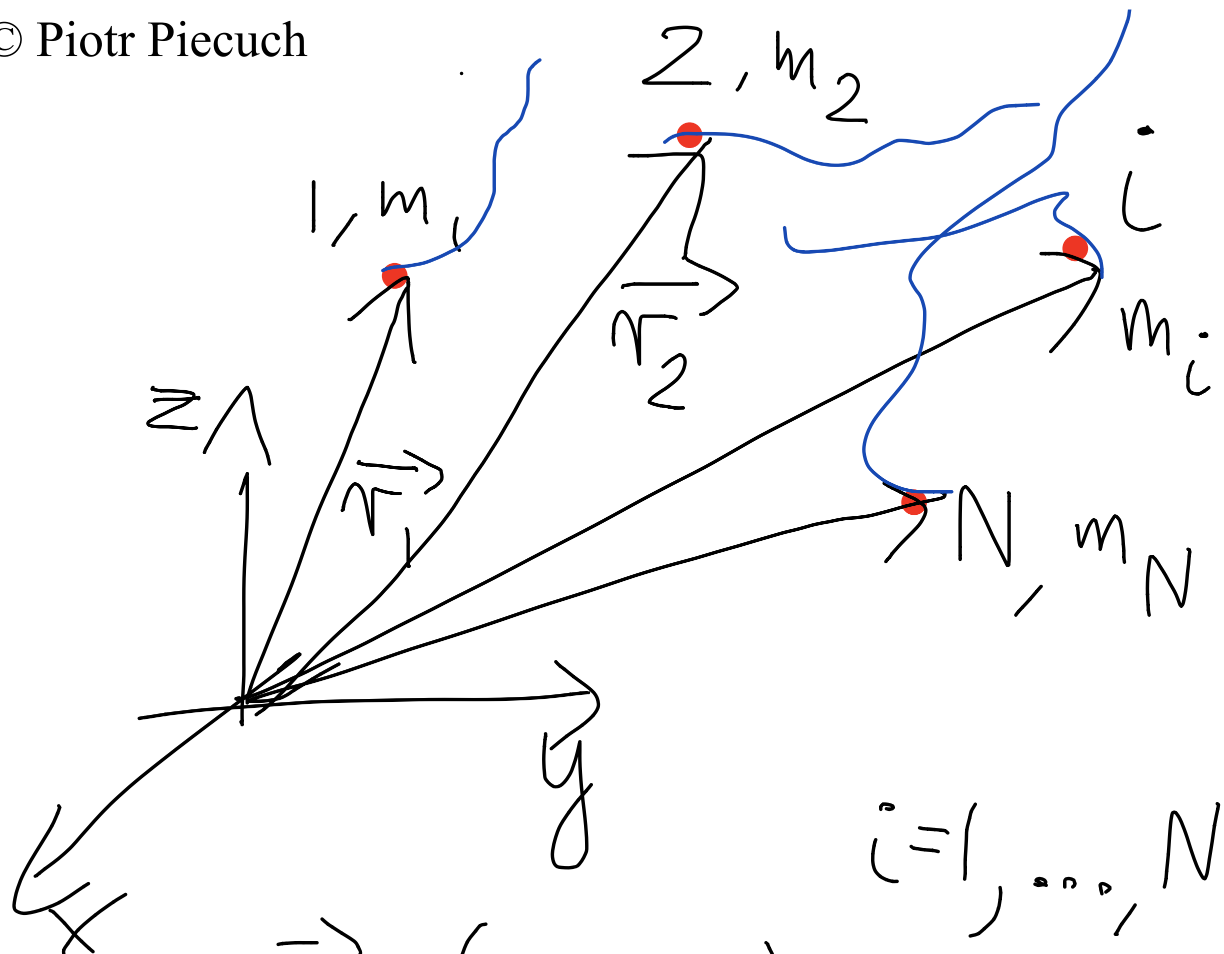


$$i = 1, \ldots, N$$

$$\vec{r}_1 = (x_1, y_1, z_1)$$

$$\vec{r}_2 = (x_2, y_2, z_2)$$

$$\vec{r}_i = (x_i, y_i, z_i)$$

$$\vec{r}_N = (x_N, y_N, z_N)$$

$$\vec{r}_i(t)$$

$$m_i \frac{d^2\vec{r}_i}{dt^2} = \vec{F}_i, \quad i=1,\dots,N$$

$$\vec{F}_i = (F_{x_i}, F_{y_i}, F_{z_i})$$

$$\vec{F}_i = \vec{F}_i(\vec{r}_1, \dots, \vec{r}_N, \dot{\vec{r}}_1, \dots, \dot{\vec{r}}_N, t)$$

$$\dot{\vec{r}}_i = \frac{d\vec{r}_i}{dt}, \quad \ddot{\vec{r}}_i = \frac{d^2\vec{r}_i}{dt^2}$$

$$m_i \ddot{\vec{r}}_i = \vec{F}_i$$

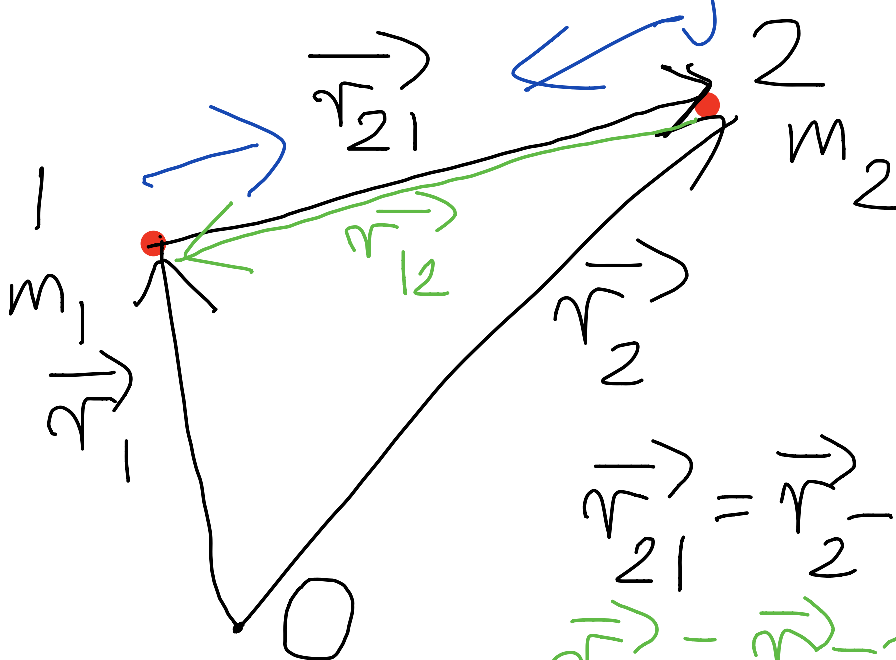
$$i = 1, \ldots, N$$

$$\vec{r}_{i0} = \vec{r}_i(t_0)$$

$$\vec{v}_{i0} = \dot{\vec{r}}_{i0} = \dot{\vec{r}}_i(t_0)$$

$$i = 1, \ldots, N$$

Gravitational force



$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$$

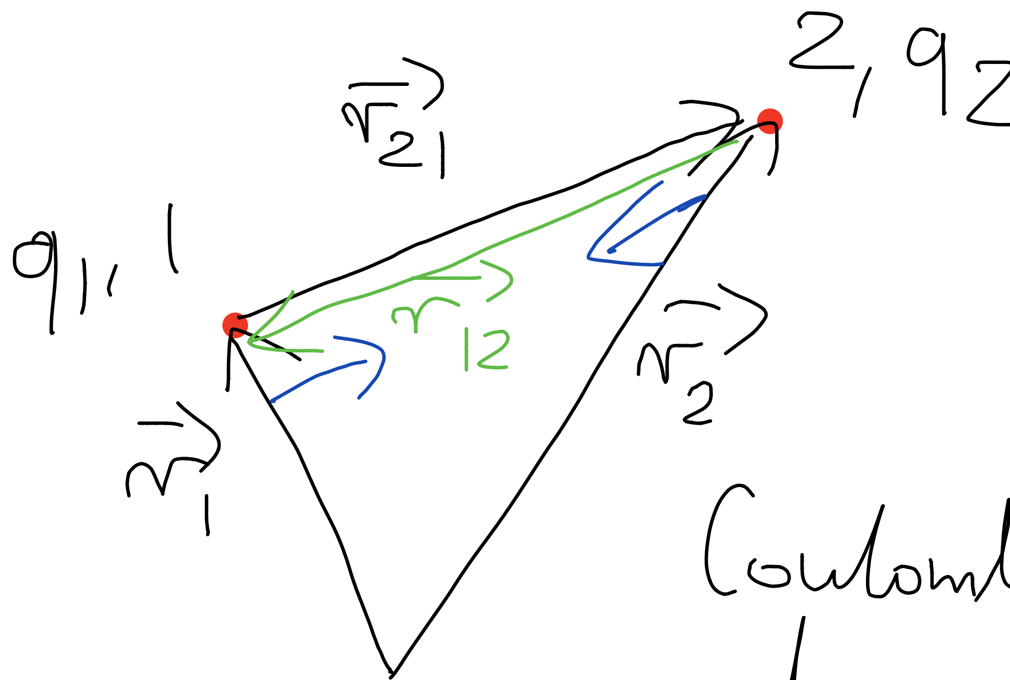
$$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$

$$\vec{F}_{21} = -\vec{F}_{12}$$

$$= -G\frac{m_1 m_2}{r_{12}^2}\frac{\vec{r}_{12}}{r_{12}}$$

on 1

due to 2



Coulomb law

$$\vec{F}_{21} = -\vec{F}_{12}$$

$$= + \frac{q_1 q_2}{r_{12}^2} \frac{\vec{r}_{12}}{r_{12}}$$

$q_1 q_2 > 0$

$q_1 q_2 < 0$

Lorentz force

$$\vec{E} = \vec{E}(\overbrace{x, y, z}^{\vec{r}}, t)$$

$$\vec{B} = \vec{B}(\overbrace{x, y, z}^{\vec{r}}, t)$$

Maxwell

$$\vec{F} = q\left[\vec{E} + \frac{1}{c}\underbrace{\vec{v}}_{= \dot{\vec{r}}} \times \vec{B}\right]$$

$$m_i \ddot{\vec{r}}_i = \vec{F}_i \quad \Big/ \quad \sum_{i=1}^{N}$$

$$\sum_{i=1}^{N} m_i \ddot{\vec{r}}_i = \sum_{i=1}^{N} \vec{F}_i$$

$$\vec{F}^{ext}$$

$$\sum_{i=1}^{N} \vec{F}_i = \sum_{i=1}^{N} \vec{F}_i^{ext} \quad /\!/$$

$$+ \sum_{i=1}^{N} \sum_{j(\neq i)}^{N} \vec{F}_{ji}$$

$$\sum_i \vec{F}_i = \vec{F}^{ext} + \frac{1}{2}\left(\sum_{i,j} \vec{F}_{ij} + \underbrace{\sum_{j,i} \vec{F}_{ij}}_{\sum_{i,j} \vec{F}_{ji}}\right)$$

$$= \vec{F}^{ext} + \frac{1}{2}\sum_{i,j}\left(\underbrace{\vec{F}_{ij} + \vec{F}_{ji}}_{=0}\right)$$

$$\sum_{i=1}^{N} m_i \ddot{\vec{r}}_i = \vec{F}^{ext}$$

$$\frac{d}{dt} \sum_{i=1}^{N} \left(m_i \dot{\vec{r}}_i \right) = \vec{F}^{ext}$$

$$m_i \dot{\vec{r}}_i = \vec{P}_i$$

$$\frac{d}{dt} \underbrace{\left(\sum_{i=1}^{N} \vec{P}_i \right)}_{\vec{P}} = \vec{F}^{ext}$$

$$\boxed{\frac{d\vec{P}}{dt} = \vec{F}^{ext}}$$

If $\vec{F}^{ext} = 0 \Rightarrow$

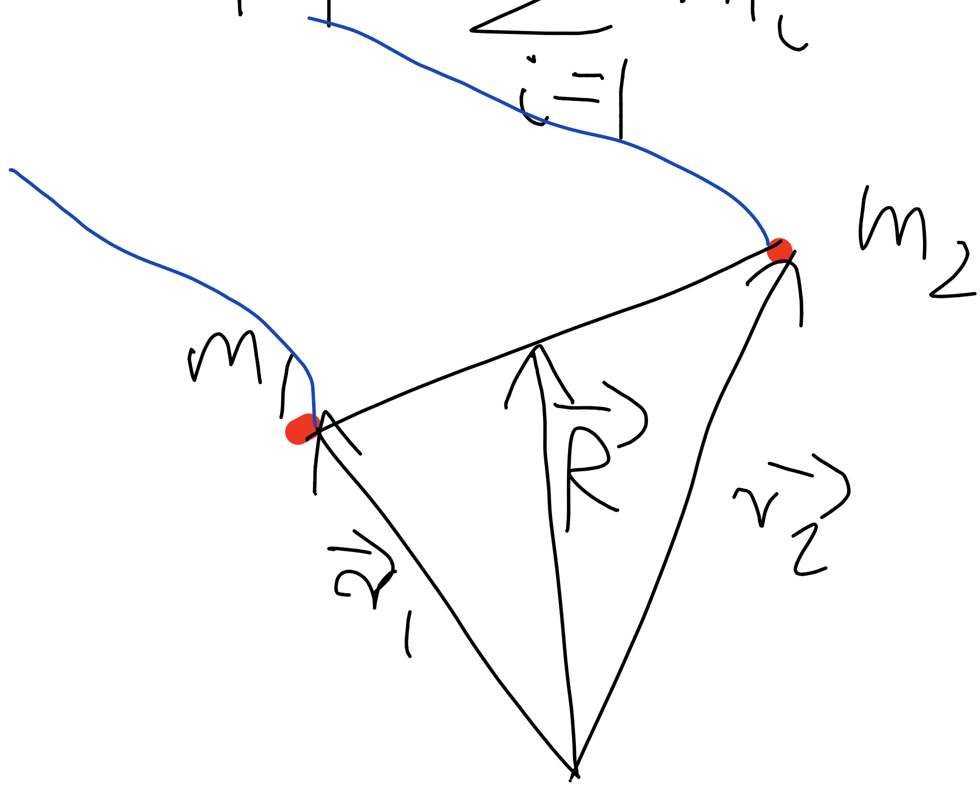
$$\frac{d\vec{P}}{dt} = 0 \Rightarrow$$

$$\Rightarrow \vec{P} = \text{const}$$

$$\sum_{i=1}^{N} m_i \dot{\vec{r}}_i = \sum_{i=1}^{N} \vec{p}_i = \text{const.}$$

$$\vec{R} = \frac{\sum_{i=1}^{N} m_i \vec{r_i}}{M} \quad \text{COM}$$

$$M = \sum_{i=1}^{N} m_i$$



$$\vec{\dot{R}} = \frac{\sum_{i=1}^{N} m_i \vec{\dot{r}}_i}{M}$$

$$M\vec{\dot{R}} = \sum_{i=1}^{N} m_i \vec{\dot{r}}_i = \vec{P}$$

If $\vec{F}^{ext} = 0$

$\Rightarrow \vec{P} = \text{const}$

$\Rightarrow \vec{\dot{R}} = \text{const.}$

$$\frac{d\vec{R}}{dt} = \text{const.}$$

$$\vec{R}(t) = \vec{R}(t_0) + \dot{\vec{R}}(t_0)(t - t_0)$$

$$\frac{d\vec{L}}{dt} = \vec{\Gamma}$$

$$\vec{D} = \sum_{i=1}^{N} \left(\vec{r}_i \times \vec{F}_i \right)$$

$$\vec{L} = \sum_{i=1}^{N} \left(\vec{r}_i \times \vec{p}_i \right)$$

$$\vec{F}_i = \vec{F}_i^{ext} + \sum_j \vec{F}_{ji}$$

$$\vec{D} = \sum_{i=1}^{N} \left(\vec{r}_i \times \vec{F}_i^{ext}\right)$$

$$+ \sum_{i,j=1}^{N} \left(\vec{r}_i \times \vec{F}_{ji}\right)$$

$$= \vec{D}^{ext} + \frac{1}{2}\left(\sum_{i,j} \left(\vec{r}_i \times \vec{F}_{ji}\right) + \sum_{\substack{j,i \\ \sim \\ i,j}} \left(\vec{r}_j \times \underbrace{\vec{F}_{ij}}_{= \left(-\vec{F}_{ji}\right)}\right)\right)$$

$$\vec{D} = \vec{D}^{ext} + \cancelto{0}{\frac{1}{2}\sum_{ij} \underbrace{(\vec{r}_i - \vec{r}_j)}_{\vec{r}_{ij} \,=\, -\vec{r}_{ji}} \times \vec{F}_{ji}}$$

$$\boxed{\vec{F}_{ji} \parallel \vec{r}_{ji}}$$

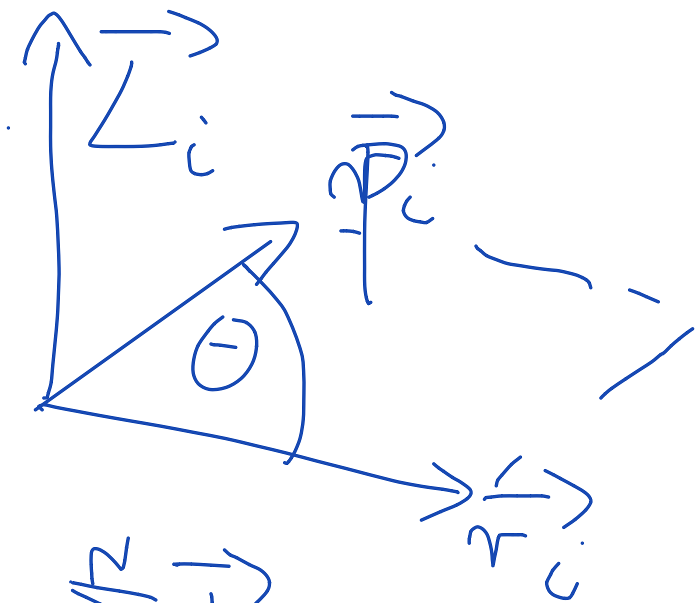
$$\frac{d\vec{L}}{dt} = \vec{D} = \vec{D}^{ext}$$

$$= \sum_{i=1}^{N} \vec{r}_i \times \vec{F}_i^{ext}$$

If $\vec{D}^{ext} = 0 \Rightarrow$

$$\Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{const}$$

$$\vec{L} = \sum_{i=1}^{N} \vec{r}_i \times \vec{p}_i$$



$$\vec{L} = \sum_{i=1}^{N} \vec{L}_i$$

$$\|\vec{L}_i\| = L_i = r_i p_i \times \sin\theta$$

$$m_i \ddot{\vec{r}}_i = \vec{F}_i, \quad i = 1, \dots, N$$

$$m_j \ddot{x}_j = X_j, \quad j = 1, \dots, 3N$$

$$\dot{x}_j \sum_j$$

$$\boxed{\sum_{j=1}^{3N} m_j \dot{x}_j \ddot{x}_j}$$

$$= \sum_{j=1}^{3N} X_j \dot{x}_j$$

$$\boxed{\sum_{j=1}^{3N} m_j \dot{x}_j \ddot{x}_j} \to T$$

$$\frac{1}{2}\frac{d}{dt}\left(\dot{x}_j^{2}\right)$$

$$= \frac{1}{\cancel{2}}\,\cancel{2}\,\dot{x}_j \ddot{x}_j = \dot{x}_j \ddot{x}_j$$

$$\frac{1}{2}\frac{d}{dt}\left[\sum_{j=1}^{3N} m_j \left(\dot{x}_j\right)^2\right]$$

$T \rightarrow$ kinetic energy

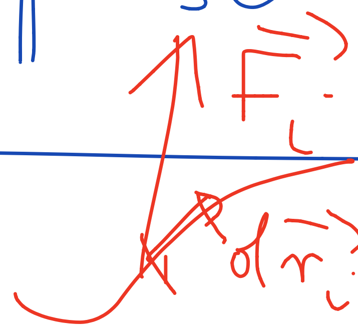
$$T = \frac{1}{2} \sum_{j=1}^{3N} m_j \dot{x}_j^2$$

$$= \frac{1}{2} \sum_{i=1}^{N} m_i \left(\dot{\vec{r}}_i\right)^2$$

$$\frac{dT}{dt} = \sum_{j=1}^{3N} X_j \dot{x}_j = \sum_{i=1}^{N} F_i \cdot \dot{\vec{r}}_i$$

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$= \|\vec{a}\|\,\|\vec{b}\|\cos\theta$$

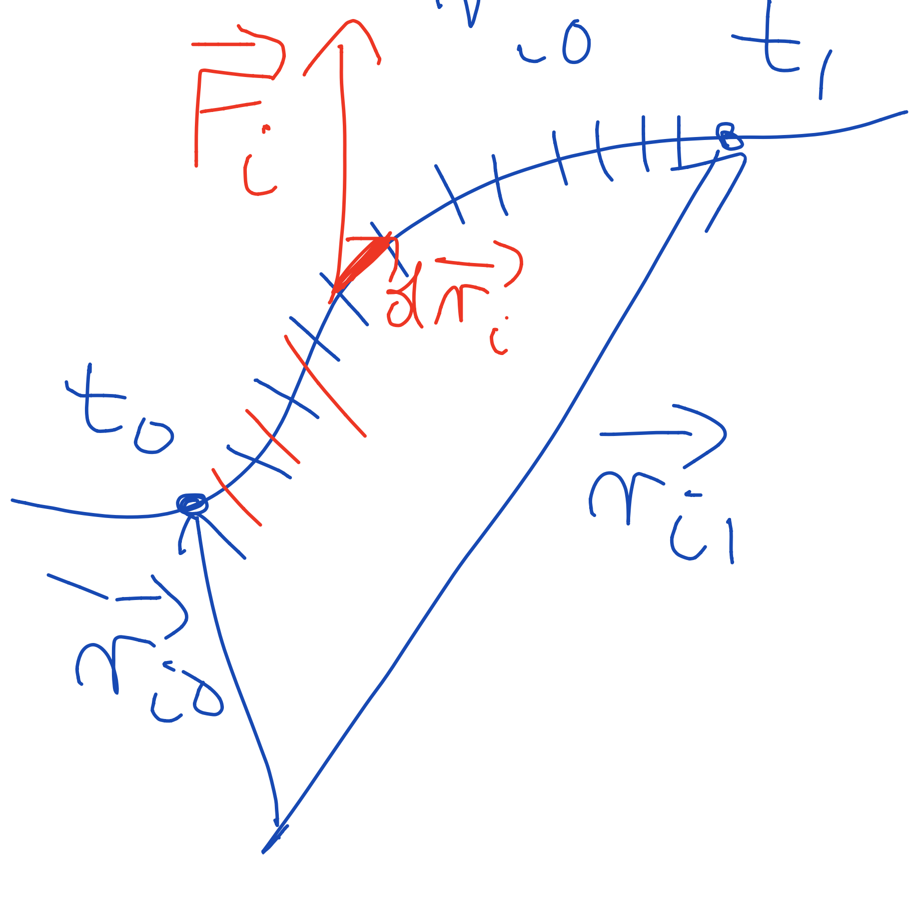


$$\int_{t_0}^{t_1} \frac{dT}{dt}\,dt =$$

$x_i \dot{x}_i\, dt$

$$= \sum_{i=1}^{N} \int_{t_0}^{t_1} \underbrace{\left(\vec{F}_i \cdot \overbrace{\dot{\vec{r}}_i\right) dt}^{d\vec{r}_i}}_{\text{work}}$$

$$T(t_1) - T(t_0) = \sum_{i=1}^{N} \underbrace{\int_{\vec{r}_{i0}}^{\vec{r}_{i1}} \vec{F}_i \cdot d\vec{r}_i}_{\text{work}}$$



derived from potential

Assume that our forces irrotational

$$V = V(\vec{r}_1(t), \ldots, \vec{r}_N(t), t)$$

potential function

$$\boxed{-\nabla_i V = \vec{F}_i}$$

$$\nabla_i = \left(\frac{\partial}{\partial x_i}, \frac{\partial}{\partial y_i}, \frac{\partial}{\partial z_i}\right).$$

$$\vec{F}_i = -\nabla_i V$$

$$\boxed{X_j = -\frac{\partial V}{\partial x_j} \quad j = 1, \ldots, 3N}$$

$$\frac{dV}{dt} = \sum_{j=1}^{3N} \frac{\partial V}{\partial x_j} \frac{dx_j}{dt} + \frac{\partial V}{\partial t}$$

Assume that

$$\boxed{\frac{\partial V}{\partial t} = 0}$$

$$\boxed{\text{conservative}}$$

$$\frac{dV}{dt} = \sum_{j=1}^{3N} \boxed{\frac{\partial V}{\partial x_j}} \dot{x}_j$$

$$= -\sum_{j=1}^{3N} X_j \dot{x}_j$$

$$\text{work} = \sum_{j=1}^{3N} \int_{t_0}^{t_1} X_j \dot{x}_j \, dt$$

$$= -\int_{t_0}^{t_1} \frac{dV}{dt} \, dt$$

$$= -\left[V(t_1) - V(t_0) \right]$$

$$= T(t_1) - T(t_0)$$

$$T(t_0) + V(t_0)$$

$$= T(t_1) + V(t_1)$$

$$= \text{const.}$$

$$= E$$

energy

$$E = T + V$$

$$V(t_1) - V(t_0) \qquad \vec{F}_i \cdot d\vec{r}_i$$

$$= -\sum_{i=1}^{N} \underbrace{\int_{t_0}^{t_1} \vec{F}_i \cdot \dot{\vec{r}}_i \, dt}_{\text{work}}$$