

Ch. 7: 4, 6, 8    Ch. 8: 1, 9, 10

- 7.4 Estimate voltage for Geiger region of operation  
 3 excited atoms for every ion pair  
 De-excitation photons have prob. of  $10^{-5}$  of creating additional avalanche

Details from problem 6.3

Anode radius = 0.003 cm

Cathode radius = 1.0 cm

P-10 gas @ 1 atm

Gas multiplication Factor?

$n'_0 = 3$ ,  $p = 10^{-5}$

Solve for  $M$  (multiplication factor) using condition of criticality for G-M

$$M n'_0 p \geq 1$$

$$M(3)(10^{-5}) \geq 1$$

$$M \approx 10^5$$

Using...

$$\ln M = \frac{V}{\ln(b/a)} \cdot \frac{\ln 2}{\Delta V} \left( \ln \frac{V}{p a \ln(b/a)} - \ln K \right)$$

$$M = 10^5$$

$$b = 1.0 \text{ cm}$$

$$a = 0.003 \text{ cm}$$

$$\Delta V = 23.6 \text{ V}$$

$$p = 1 \text{ atm}$$

$$K = 4.8 \times 10^4 \text{ V/cm} \cdot \text{atm}$$

$$\ln M = \frac{V}{\ln(b/a)} \left( \frac{\ln 2}{\Delta V} \right) \ln \left( \frac{V}{p a \ln(b/a) K} \right)$$

$$V \ln V = \frac{\ln(b/a) \ln M}{\ln 2} \Delta V + \ln(p a \ln(b/a) K)$$

$$V \ln V = 2277.1 + 6.7 = \cancel{2283.8} \quad \text{Math?}$$

$$\boxed{V = 383.8}$$

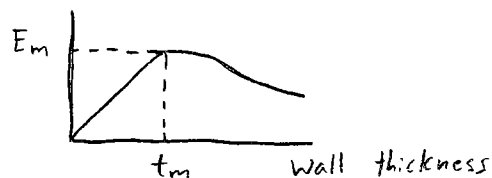
$$\ln(3.3 \times 10^4) = \frac{V_0}{\ln(1/0.003)} \left( \frac{\ln 2}{23.6} \right) \ln \left( \frac{V_0}{1 (0.003) \ln \frac{1}{0.003} 4.8 \times 10^4} \right)$$

$$10.40 = V_0 (5.056 \times 10^{-3}) \ln(V_0 / 836.5)$$

$$2.05 \times 10^3 = V_0 [\ln V_0 - \ln(836.5)]$$

Above  $\rightarrow V_0 = 2160$  Volts

## 7.6 Medium Energy Gamma Rays (1 MeV)



a) The shape of the wall can be explained by the gamma-ray interactions with the wall of the container. The gamma-ray counting efficiency rises to a maximum at  $t_m$  because if the wall is too thin, the gamma-ray may not interact with the wall, but pass through it. The efficiency decreases as the thickness is increased beyond  $t_m$  because the electrons produced in the interaction may be stopped before reaching the gas. ✓

b) Assume 1 MeV  $e^-$  → Typical range of  $e^-$  in moderate density materials is approximately in mm/MeV  
Estimated order of magnitude for wall thickness:  $10^{-3}$  m ✓

7.8  $p = 0.5 \text{ atm}$   
 $\mu = 1.5 \times 10^{-4} \text{ m}^2 \cdot \text{atm} / \text{V} \cdot \text{s}$   
 Anode radius =  $0.005 \text{ cm}$   
 Cathode radius =  $2 \text{ cm}$   
 $V_{\text{app}} = 1500 \text{ V}$   
 $E_{\text{thresh}} = 2 \times 10^6 \text{ V/m}$

$$E(r) = \frac{V}{r \ln(b/a)}$$

$$E_{\text{thresh}} = \frac{V}{r \ln(b/a)}$$

$$2 \times 10^6 \text{ V/m} = \frac{1500 \text{ V}}{r \ln(2/0.005)}$$

$$r = 1.25 \times 10^{-4} \text{ m}$$

$v =$  drift velocity

$$v = \frac{\mu E_{\text{thresh}}}{p}$$

$$v = \frac{(1.5 \times 10^{-4} \text{ m}^2 \cdot \text{atm} / \text{V} \cdot \text{s})(2 \times 10^6 \text{ V/m})}{0.5 \text{ atm}}$$

$$v = 600 \text{ m/s}$$

distance from cathode to multiplying region

$$(2 \text{ cm} - 0.0125 \text{ cm}) = 1.9875 \text{ cm} = 0.019875 \text{ m}$$

$$0.019875 \text{ m} / (600 \text{ m/s}) = 3.315 \times 10^{-5} \text{ s}$$

$\rightarrow 33 \mu\text{s}$  but this is too short for reason above

the real problem is that you need average field



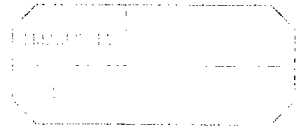
$$\text{drift time} = \frac{\Delta S}{\frac{\mu}{p} \bar{E}} = \frac{(b - r_c)}{\frac{\mu}{p} \frac{V_0}{\ln \frac{b}{a}} [\text{Inst}]}$$

$$\bar{E} = \frac{\int_{r_c}^b V_0 \frac{1}{r} dr}{\int_{r_c}^b \frac{1}{r} dr} \quad \text{3.23}$$

$$\text{time} = \frac{(b - r_c)}{\left(\frac{\mu}{p}\right) \frac{V_0}{\ln \frac{b}{a}} (\ln b - \ln r_c)}$$

$$\int \frac{1}{r} dr = \ln r$$

$$\begin{aligned} \text{time} &= \frac{(0.02 - 1.25 \times 10^{-4})^2 \ln(0.02 / 5 \times 10^{-5})}{\left(\frac{1.5 \times 10^{-4} \frac{\text{m}^2 \text{ atm}}{\text{V} \cdot \text{sec}}}{0.5 \text{ atm}}\right) 1500 \text{ V} (\ln 0.02 - \ln 1.25 \times 10^{-4})} \\ &= 1.0 \text{ msec} \end{aligned}$$



8.1 Scintillation eff.

1 MeV of particle energy loss  $\rightarrow$  20,300 photons

$$\lambda = 447 \text{ nm}$$

$$c = \lambda \nu$$

$$\nu = 6.7 \times 10^{14} \text{ s}^{-1}$$

$$E = h\nu$$

$$E = 4.45 \times 10^{-19} \text{ J} \Rightarrow 2.78 \text{ eV}$$

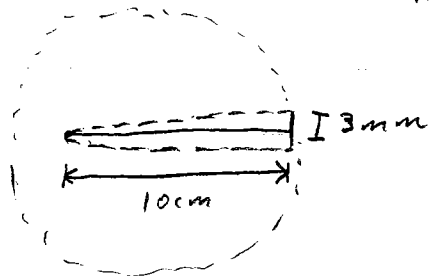
$$20,300 \text{ photons} \times \frac{2.78 \text{ eV}}{\text{photon}} = 56434 \text{ eV}$$

$$\frac{56434 \text{ eV}}{1 \times 10^6 \text{ eV}} = 0.056 \rightarrow \boxed{5.6\%} \checkmark$$

8.9

1 MeV  $\beta$  in NaI(Tl)

can detect 10 visible photons

Light yield for NaI(Tl)  $\rightarrow$  1 MeV produces 38,000 photons

$$\text{Surface area} = 4\pi r^2 = 4\pi(0.1\text{ m})^2 = 0.126\text{ m}^2$$

$$\text{Area of eye} = \pi r^2 = 7.07 \times 10^{-6}\text{ m}^2$$

$$\frac{\text{Eye Area}}{\text{Surface Area (emitted)}} = \frac{7.07 \times 10^{-6}\text{ m}^2}{0.126\text{ m}^2} = 5.6 \times 10^{-5}$$

$$\# \text{ of photons reaching eye} = (5.6 \times 10^{-5})(38,000 \text{ photons})$$

# of photons = 2.1 photons  
unable to detect

8.10

a) 1 MeV fast  $e^-$  0.3 mm diameter plastic fiber

Estimate 1mm/MeV  $\rightarrow$   $\sim$  0.3 MeV energy loss  
 from where? 2mm/MeV  $\rightarrow$   $\sim$  0.6 MeV energy loss (too large)

b) Assuming 8-10 photons/keV

And 0.3 MeV energy deposited

300keV (9  $\frac{\text{photons}}{\text{keV}}$ ) = 2700 photons  $\rightarrow$

c)  $n_{\text{core}} = 1.58$

$n_{\text{cladding}} = 1.49$

attenuation length = 2m

# of scint. photons arriving at one end of fiber  
 1m from point of interaction

$$F = \frac{1}{2} \left( 1 - \frac{n_1}{n_0} \right)$$

$$F = \frac{1}{2} \left( 1 - \frac{1.49}{1.58} \right)$$

$$F = 0.0285$$

$n_1$  = refractive indices of cladding  
 $n_0$  = refractive indices of core

Fraction of rays captured in one direction

$$\frac{I}{I_0} = e^{-x/L}$$

$$\frac{I}{I_0} = e^{-1/2}$$

$$\frac{I}{I_0} = 0.607$$

$x$  = distance from point of interaction  
 $L$  = attenuation length

$$(0.0285)(0.607)(2700 \text{ photons}) = \boxed{46.7 \text{ photons}}$$