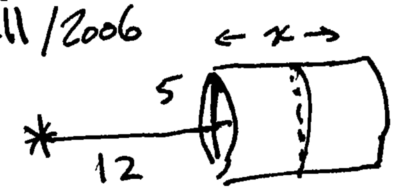
Chem 988
Fall/2006

1)



$$V\Omega = 750\ \text{cm}^3 = \pi r^2 x$$

$$x = \frac{750\ \text{cm}^3}{\pi\,(25\ \text{cm}^2)} = 9.549\ \text{cm}$$

(a)[5] $\epsilon_n = 1 - e^{-\rho_N \sigma x}$

$$\rho_N = 5.323\ \frac{\text{g}}{\text{cm}^3}\left(\frac{6.022\times10^{23}/\text{mole}}{72.64\ \text{g/mole}}\right) = 4.411\times10^{22}/\text{cm}^3$$

$$\epsilon_n = 1 - e^{-4.411\times10^{22}(4\times10^{-24})\,9.549}$$

upper limit due to approximate geometry

$$\epsilon_n = 1 - e^{-1.685} = 1 - 0.185 = 0.815\ ,\quad \epsilon_n \sim 0.8\ \left[\sigma \text{ only has } 1 \text{ sig. fig}\right]$$

(b)[5] $A = \text{Rate of Production}\left(1 - e^{-\lambda t_{\text{Bombard}}}\right)$ but $t_{\text{Bomb}} \gg t_{1/2}$ so $e^{-\lambda t_B} \to \emptyset$

$$A = \text{Rate of Prod} = N_0 \sigma \phi$$

$$N_0 = \rho_N x * 0.38 = 1.601\times10^{23}/\text{cm}^2$$

$$\phi_1 = \text{Rate of emission} * \frac{\Delta\Omega}{4\pi}$$

$$\phi_1 = (2\times10^5/\text{s})\ \frac{1}{2}\underbrace{\left[1 - \frac{12}{\sqrt{12^2+5^2}}\right]}_{0.03846}$$

$$\phi_1 = 7692/\text{s}$$

$$A_1 = 1.601\times10^{23}(1\times10^{-24})\,7692.$$

$A_1 = 1231$/s

$A_1 \sim 1000$ Bq [only 1 sig fig]

Better to take ½-point of detector due to close geometry

$$\phi_2 = (2\times10^5/\text{s})\ \frac{1}{2}\underbrace{\left[1 - \frac{(12+4.78)}{\sqrt{16.78^2+5^2}}\right]}_{0.02082}$$

$$\phi_2 = 4164/\text{s}$$

$A_2 = 667$/s

$A_2 \sim 700$ Bq

(c)[5] $\epsilon_n = 1 - e^{-\rho_N \sigma x}$

for plastic $x = 2.54$ cm

manufacturer says it (BC-400) is vinyltoluene, C_9H_{10}

$$\rho_N = 1.032\ \text{g/cm}^3\ \frac{6.022\times10^{23}/\text{mole}}{118.07\ \text{g/mole}} = 5.264\times10^{21}\ \frac{\text{molecules}}{\text{cm}^3}$$

c - continue $\epsilon_N = 1 - e^{-\rho_N \sigma x}$ $\quad x = 2.54$ cm

$\rho_N = 5.264 \times 10^{21}/cm^3$ for C_9H_{10}

$\sigma(E = 1.0\ MeV)$ for ${}^{12}C$, ${}^{1}H$ from the text fig 15.15

$$\sigma_{molecule} = 9 \times \sigma({}^{12}C) + 10\ \sigma({}^{1}H) = 9(2.5\ b) + 10(4\ b)$$

$$= 62.5\ b$$

$$\epsilon_N = 1 - e^{-\overbrace{5.264\times10^{21}/cm^3 \left(62.5\times10^{-24}\ cm^2\right)}^{0.329/cm} 2.54\ cm}$$

$$\epsilon_N = 1 - e^{-0.8356} = 1 - 0.4336 = 0.5664$$

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2) ${}^{241}_{95}Am \rightarrow {}^{4}He + {}^{237}_{93}Np$ $\quad E_\alpha = \dfrac{5485\ keV \quad 84.5\%}{5443\ keV \quad 13.0\%}$ $\left\{\text{Table of the Isotopes}\right.$

10

(max E_n comes from higher E_α)

$${}^{4}He + {}^{9}Be \rightarrow ({}^{13}C^*) \rightarrow {}^{12}C + {}^{1}n + Q$$ $\left\{\text{AMDC Masses}\right.$

$\Delta = 2424.9 \quad 11347.6 \qquad 0 \quad 8071.3$ keV

$$Q = (2424.9 + 11347.6 - 8071.3)\ keV = \underline{\underline{5701.2\ keV}}$$

(max E_n comes for $\theta_n = 0^\circ_{LAB} = 0^\circ_{CMS}$)

$$E_\alpha + 0 = E_C + E_n + Q$$

$$p_\alpha^2/2m_\alpha = p_C^2/2m_C + p_n^2/2m_n + Q$$

take $m_\alpha \cong 4m_n$, $m_C \cong 12 m_n$

$$p_\alpha^2/4 = p_C^2/12 + p_n^2 + 2m_n Q$$

$$p_\alpha^2/4 = (p_\alpha - p_n)^2/12 + p_n^2 + 2m_n Q$$

$m_n = 939\ \frac{MeV}{c^2}$ $\quad p_\alpha = 202.2\ \frac{MeV}{c}$ $\quad Q = 5.701\ MeV$

$p_\alpha = p_n + p_C \rightarrow p_C = p_\alpha - p_n$

$p_\alpha \sim \sqrt{2 m_\alpha E_\alpha}$

$p_\alpha \sim \left(2\times4\times931.5\frac{MeV}{c^2}\ 5.485\ MeV\right)^{1/2}$

$p_\alpha \sim 202.2\ MeV/c$

(some math)

$p_n = 138.\ MeV/c \rightarrow E_n = 10.2\ MeV$

3) (a) 5

- one detector fires on γ_1
- 11 detectors are available to observe γ_2

$$\epsilon = 11\left(\frac{0.02 \text{ photopeak total}}{12 \text{ detectors}}\right) = 0.018$$

(b) 5 before summing losses $R = 100/s \times 0.02 = 2/s$

Random summing looks like it could be a problem...

$r_{1pu} = (r_1 \tau)\, r_{pu}$ from notes or eq 10.13 in text

$$r_{1+pu} = \left(100/s \times 500 \times 10^{-9}s\right) 5000/s = 0.25/s$$

→ not a serious problem $\quad R = (100 - 0.25) \times 0.02 = 2/s$

(c) 10 calculate deadtime per event at a rate of 5000×2 words/s

fraction dead = $R_{OBS}\,\tau$, assume digiting in parallel

① $\tau = \dfrac{\langle \text{average channel} \rangle}{400 \times 10^6 \text{ Hz}} + (2 \times 3\mu s)$, conversion + data transfer in parallel

$$\tau \sim \frac{8192/2}{4 \times 10^8 /s} + 6\mu s = 10.2 + 6\ \mu s = 16.2\mu s$$

② $$R_{OBS} = \frac{R_{TRUE}}{1 + R_{TRUE}\,\tau} = \frac{5000}{1 + 5000 \times 16.2 \times 10^{-6}} = 4625.$$

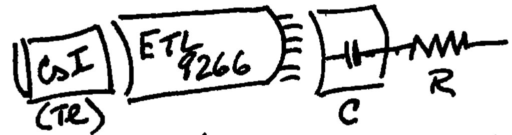
③ $$\text{fraction dead} = 4625/s \times 16.2\mu s \times 10^{-6} \frac{s}{\mu s} = 0.075$$

(d) 5 Cross talk from Compton scattering in one detector that leads to a "Backscatter" into the neighbor detector

$E \sim 1 \text{ MeV} - .256 \sim 0.75 \text{ MeV}$

$E_{BS} \sim 0.256 \text{ MeV} \sim \left(\frac{.511}{2}\right)$

4)



(a) 10 1 MeV into CsI(Tl) gives 65 k photons/MeV Table 8.3 in text

$\lambda = 540$ nm

$V = \frac{Q}{C}$ for large τ, $\tau = 100 \times 10^{-12}$ F $\times 10^{5}\,\Omega = 10\,\mu s >$ decay time of 0.68 μs & 3.3 μs

$$V = \frac{Q}{C} = \frac{(\#\text{ photons})}{C}\left(\begin{matrix}\text{Quantum}\\ \text{Efficiency}\end{matrix}\right)\left(\begin{matrix}\text{Tube}\\ \text{gain}\end{matrix}\right) \times q_e$$

Table 9.1 in text

Gain $= 0.6 \times 10^{6}$

Radiant Sens. = 100 mA/W

$$QE = \frac{R.S.\ hc}{q_e\ \lambda}$$

$[Q = 1.44\ nC]$

$$V = \frac{65000}{100 \times 10^{-12}\,F}\overbrace{\left(\frac{0.1\ A/W \quad 6.626 \times 10^{-34}\ J\cdot s \times 3 \times 10^{8}\ m/s}{q_e \quad 540 \times 10^{-9}\ m}\right)}^{3.681 \times 10^{-20}/q_e} 0.6 \times 10^{6}\ q_e$$

$V = 14.4$ V

(b) 10 Resolution will be determined by # of photo electrons ...

$$\#\text{ photo electrons} = 65000 \text{ photons } (QE)$$

$$= 65000\left(\frac{3.681 \times 10^{-20}}{1.602 \times 10^{-19}}\right) = 65000\,(0.2298)$$

$$= 14940$$

$$\sigma \sim \frac{1}{\sqrt{N}} = 8.2 \times 10^{-3} \quad \text{or } 0.82\%$$

$$\text{Resolution} = \text{FWHM} = 2.354\,\sigma = 1.9 \times 10^{-2} \quad \text{or } 1.9\%$$

5) (a) 5

$$\left(\frac{1\ GeV}{1\ Volt}\right)^{-1} \Rightarrow \frac{10^{3}\ mV}{10^{9}\ eV} \times \frac{3.62\ eV/IP \text{ in Si}}{1\ e^-/IP \quad 1.602 \times 10^{-19}\ \frac{Coul}{e^-}} \times 10^{+15}\ \frac{fC}{Coul} = 2.26 \times 10^{-2}\ \frac{mV}{fC}$$

$$\rightarrow 2.26 \times 10^{-2}\ \frac{mV}{fC} = 22.6\ \frac{mV}{pC}$$

lost conversion factor −2

5)

(b) 5 ΔE for $90 \frac{MeV}{A}$ ^{44}S in Si = 190.5 MeV from LISE++

$$\frac{\text{sig}}{\text{noise}} = \frac{190.5 \times 10^{6}\ eV / 3.62\ eV/e^-}{30000\ \text{electrons}} = 1754.$$

-2 no conversion factor

(c) 5 $90 \frac{MeV}{A}$ $^{44}S^{16+}$ $B\rho = 3.84491$ Tm $\rightsquigarrow$ $96.84 \frac{MeV}{A}$ $^{45}Cl^{17+}$

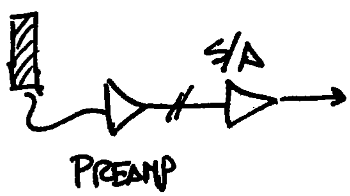
-1 not mag. rigidity

$\Delta E(Cl) = 203.79$ MeV

$$\frac{\Delta \text{sig}}{\text{noise}} = \frac{\Delta(\Delta E)}{\text{noise}} = \frac{\left[(203.79 - 190.5)/3.62\right] \times 10^{6}}{30\,000} = 122.$$

[no problem here]

(d) 5



① Preamp gives $\frac{1\ V}{1\ GeV} \times 0.1905\ GeV = 0.190\ V$

② cable attenuation 100 m RG 58 C/U

Table 16.1 $\quad 0.174\ dB/m \times 100\ m = 17.4\ dB$ @ 100 MHz

$\quad 0.413\ dB/m \times 100\ m = 41.3\ dB$ @ 400 MHz

$$1\ dB = 10 \log \frac{P_1}{P_2} \simeq 20 \log \frac{V_1}{V_2}$$

$$\rightsquigarrow V_1 = V_2\, 10^{dB/20}$$

$10^{17.4/20} = 7.41$

$10^{41.3/20} = 116.1$

(High frequency signals are gone!)

$$V_{\text{out cable}} = 0.190\ V / 7.41 = 0.0256\ V$$

③ Shaping Amp $V_{OUT} = 5V$

$$\text{Gain} = \frac{5V}{0.0256V} = 195$$

5) (e)

10

$$C\ (\text{parallel plate}) = \frac{\epsilon A}{d} = \frac{10\times 10\ \text{cm}^2}{0.3\ \text{cm}}\ 8{,}854\times 10^{-12}\ \frac{\text{coul}^2}{N/m^2}\times 10^{-2}\ \frac{m}{\text{cm}}$$

$$C = 2.95\times 10^{-11}\ F$$

$$(RC = 50\times 2.95\times 10^{-11} = 1.5\ ns)$$

$$V = \frac{Q}{C} \rightsquigarrow Q = VC = \underset{\uparrow}{2}\left[5\times 10^{-3}V\times \cancel{2.95}\times 10^{-11}F\right] = 2.95\times 10^{-13}\ \text{coul}$$

two ends resistor chains

$$\left[\hookrightarrow 1.84\times 10^{6}\ e^-\right]$$

also

$$Q = \frac{G\,\Delta E}{W} = \left(\frac{\overbrace{0.2\ \text{MeV}\times 10^{6}}^{6873\ e^-}}{29.1\ \text{eV}/e^-}\right)G$$

use Table 5.1 for Methane & α-particles

$$G = 1.84\times 10^{6}\ e^-\left(\frac{29.1}{0.2\times 10^{6}}\right)$$

$$G = 268.$$