

Week 7: Ch. 10 Spec. w/ Scintillation Ctrs.

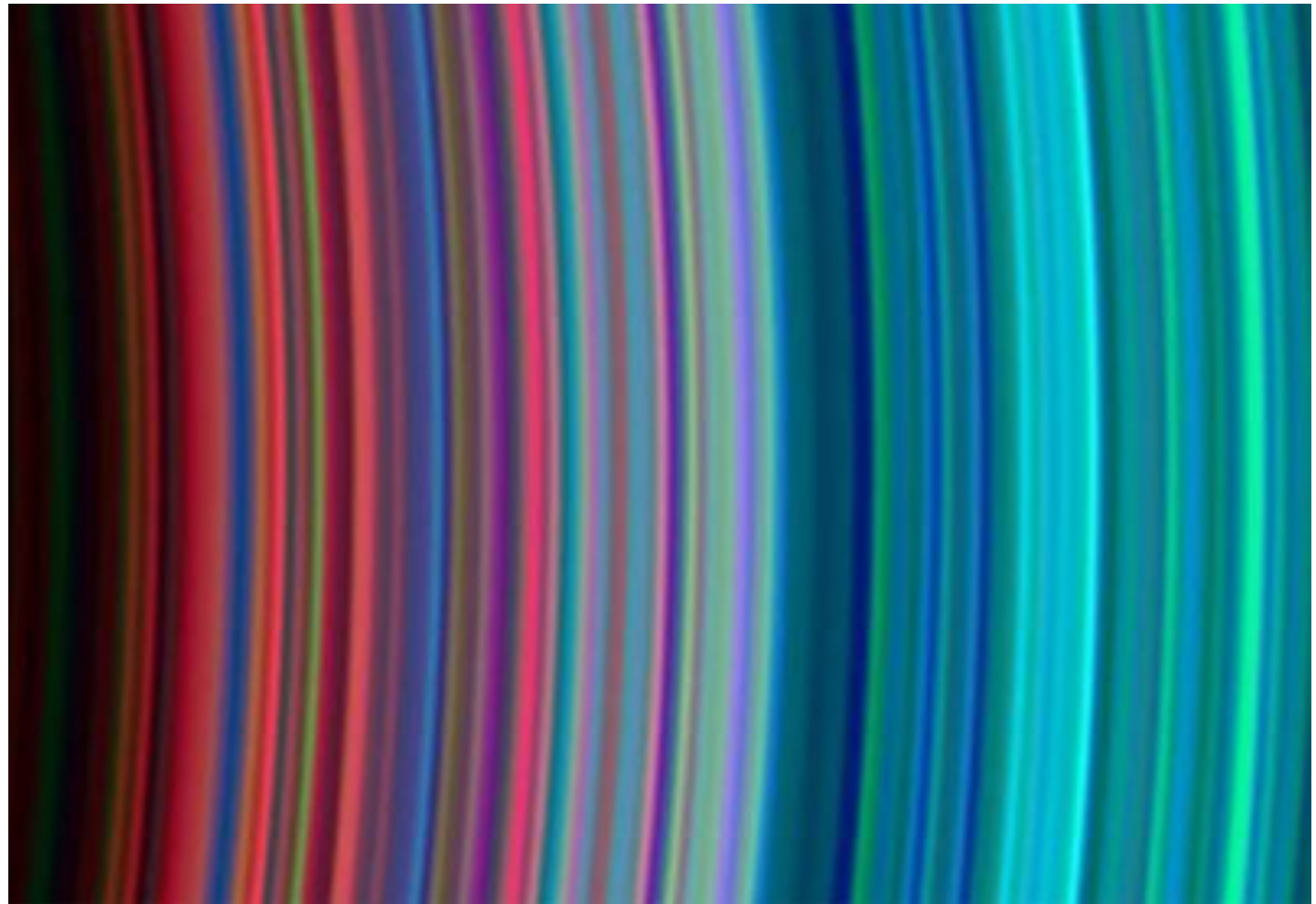
Photomultiplier Devices

Spectroscopy with Scint. Counters

- gamma-ray interactions, reprise
- observed spectra
- spectral components, backscatter
- summing
- position measurement

Semiconductor Diodes

Rings of Saturn
NASA/JPL



Chap. 10 – Spectroscopy with Scintillators

Spectroscopy implies the measurement of the *energy* of the radiation.

Charged particles: the light output was shown to be proportional to the range of the particles and so each measurement of each particle at each energy requires a calibration – limiting the applicability.

$$\frac{dL}{dx} = \frac{S}{kB} \quad \text{when} \quad \frac{dE}{dx} \text{ is large}$$

Neutrons: primarily detected with organic scintillators and interact by scattering from hydrogen or capturing on seed materials. The scattering leaves varying amounts of energy in the material (depends on kinematics of random collisions). Neutron spectroscopy with scintillators relies on Time-of-flight techniques that can often even ignore the energy deposited in scintillator.

Gamma-rays: are very penetrating and high density materials are needed to have significant absorption → inorganic scintillators.

Recall there are three classes of interactions: PE, CS and PP, and the probability of each depends on the photon energy and on the Z of the absorber.

The workhorse scintillator for photons is NaI(Tl), the observed spectra have characteristic features that depend on the incident photon energy. Recent efforts have attempted to identify new materials:

e.g., [LaBr₃\(Ce\)](#)

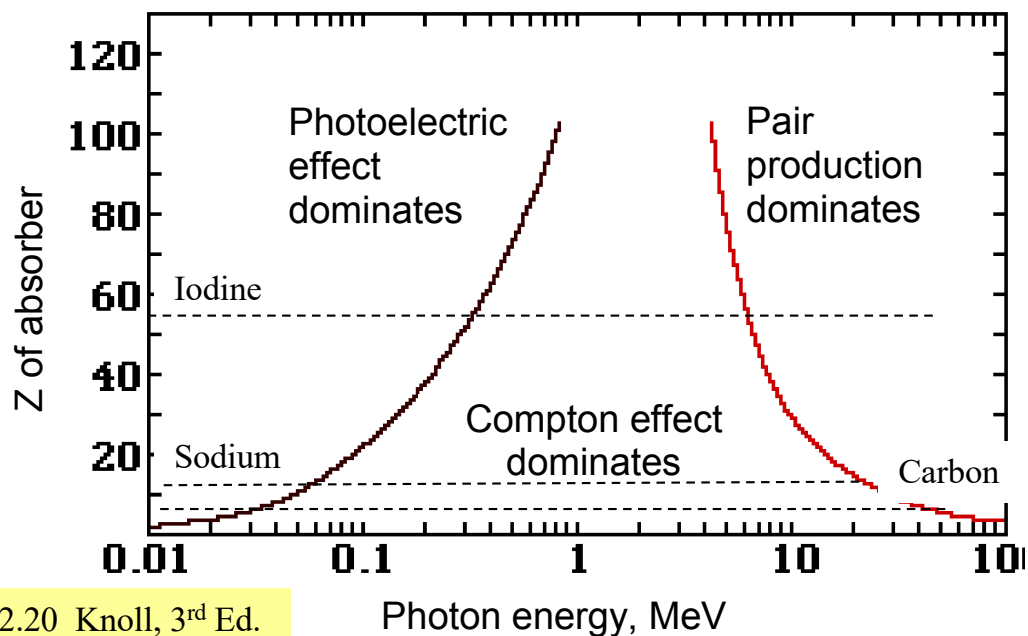


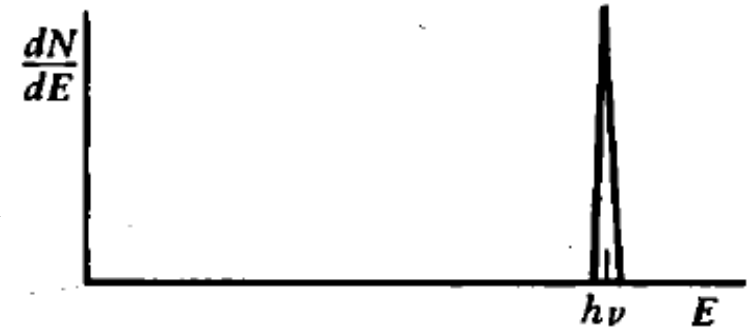
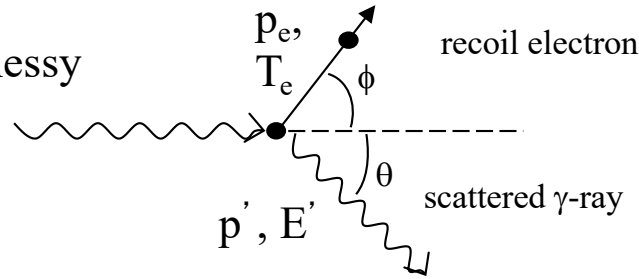
Fig. 2.20 Knoll, 3rd Ed.

Spectroscopy w/ Scintillators – Line Shape

Photoelectric Absorption: low photon energy phenomenon, complete conversion of $h\nu$ into KE of one e^- then on into visible photons, back into photoelectrons, then into a current.

Compton Scattering: messy

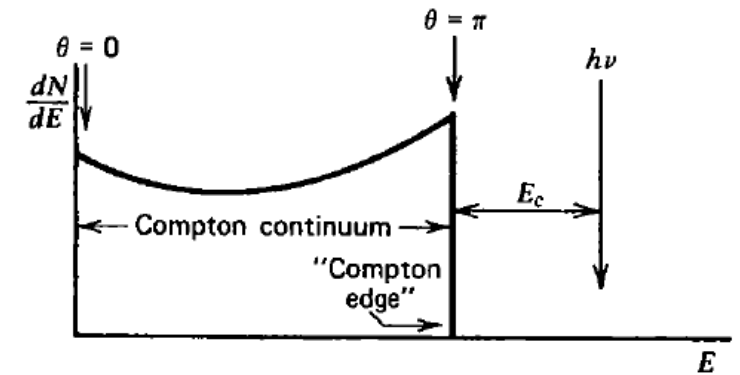
$$h\nu' = \frac{h\nu}{1 + \left(\frac{h\nu}{m_e c^2}\right)(1 - \cos\theta)}$$



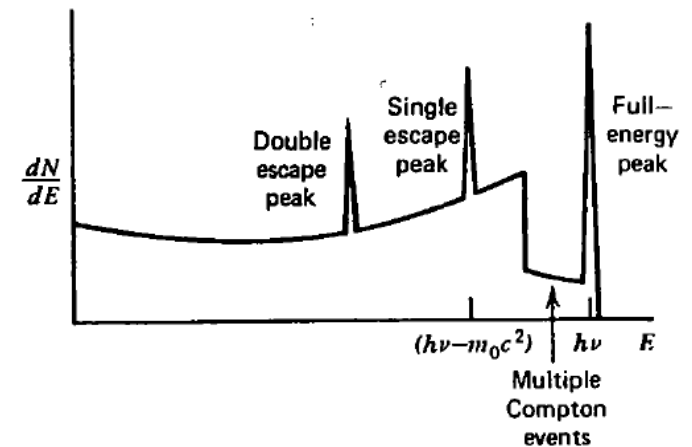
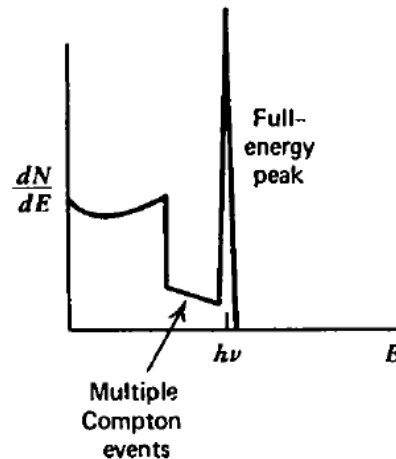
Pair production: photons with more than twice the rest mass energy of an electron can convert (in an electric field) into a positron/electron pair. The positron annihilates at the end of its range creating two new photons ($E=511$ keV) each that may escape. The fraction of escapes depends on the crystal dimensions.

$$h\nu < 2m_0c^2$$

$$h\nu \gg 2m_0c^2$$

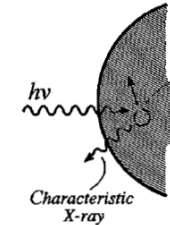
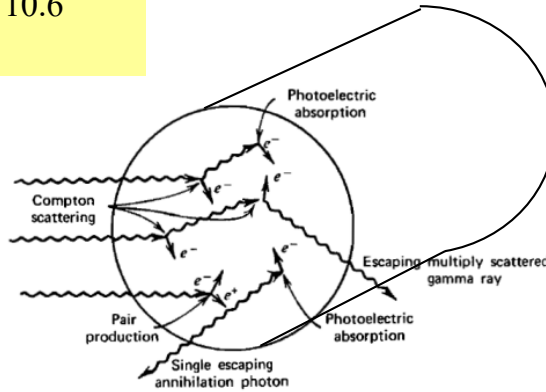
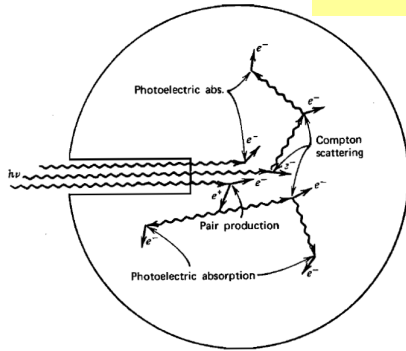


Two or three processes take place for each incident γ ray creating a complicated spectrum even from a monoenergetic source.



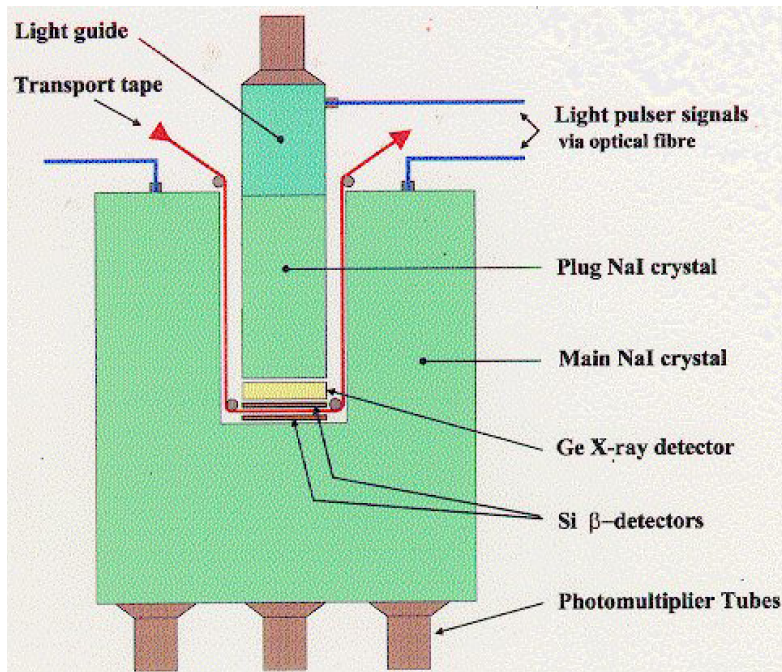
Spectroscopy w/ Scintillators – Incomplete Interactions

Figs. 10.2, 10.3, 10.4, 10.6 from Knoll, 3rd Ed.



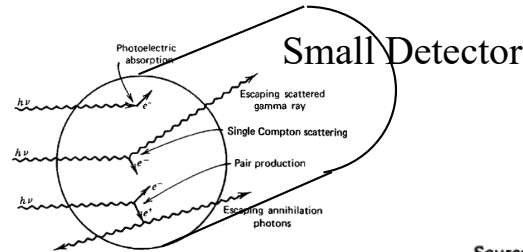
Surface interactions

Sum Spectrometer

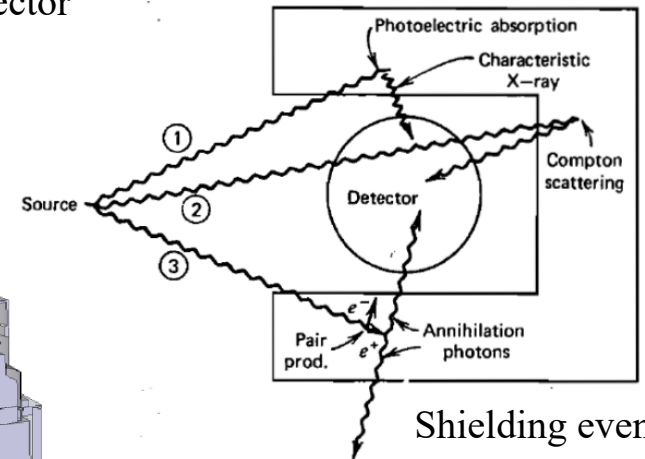


The size of the main crystal ($\varnothing 14'' \times 14''$) makes TAS one of the LARGEST single-crystal NaI(Tl) detectors in the world!

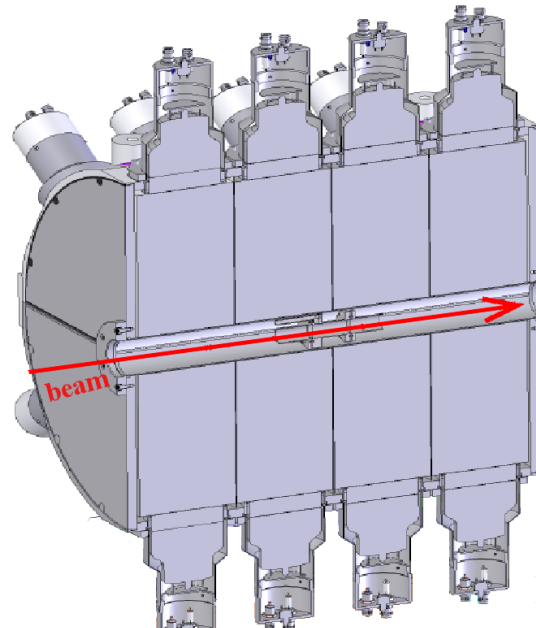
© DJMorrissey, 2019



Small Detector



Shielding events

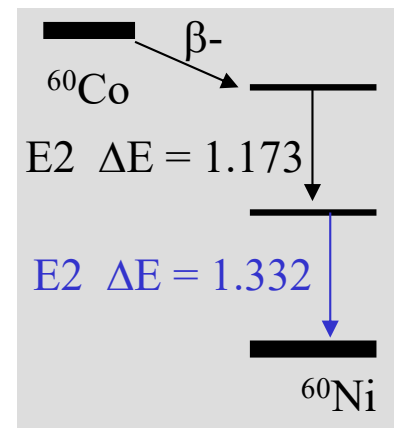
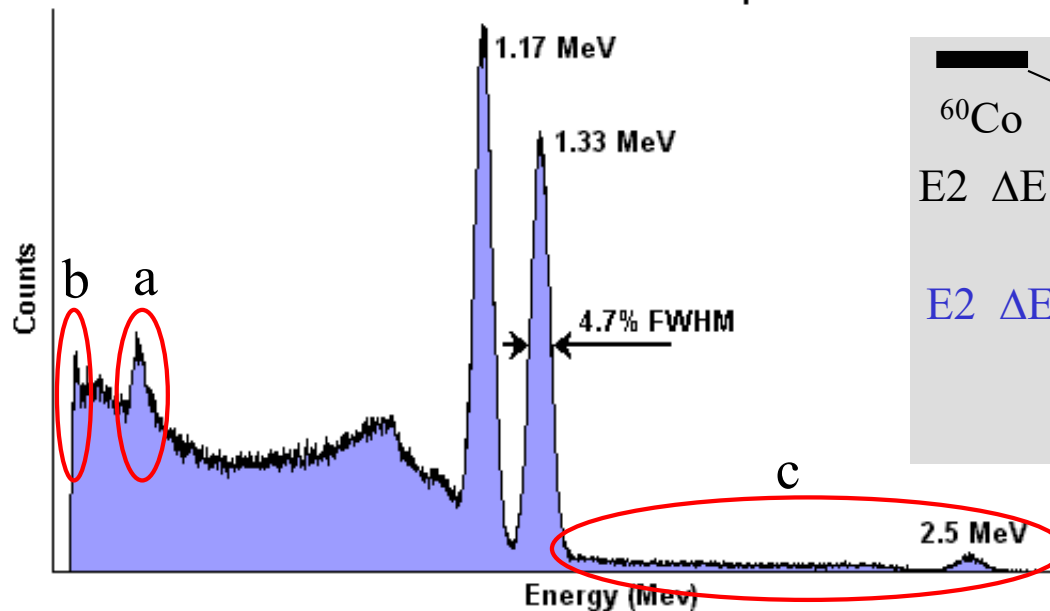


<https://groups.nsl.msu.edu/SuN/>

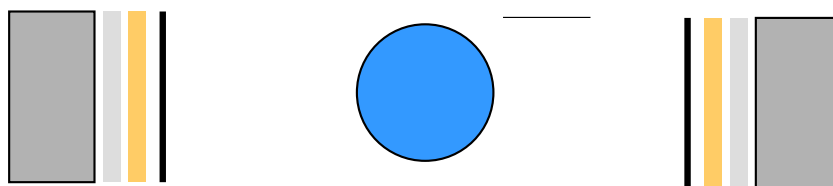
Spectroscopy w/ Scintillators – Observed Spect.

76B76 NaI Detector: ^{60}Co Spectrum

^{60}Co one expects:
two full-energy peaks,
two Compton edges ..



Compton backscatter, the photon scattered at $\theta \sim 180^\circ$ from the environment back into the detector. $E(h\nu, \theta \sim 180^\circ) \sim m_0c^2/2$
Solution: move shielding far from the detector.



Key															
atomic number		symbol		atomic weight (mean relative mass)		group		period		block		state		half-life	
1	H	3	Li	4	Be	11	Na	12	Mg	19	K	20	Ca	27	Co
2	He	5	B	6	C	7	N	8	O	9	F	10	Ne	13	Al
13	Al	14	Si	15	P	16	S	17	Cl	18	Ar	21	Sc	22	Ti
23	V	24	Cr	25	Mn	26	Fe	28	Ni	29	Cu	30	Zn	31	Ga
31	Ga	32	Ge	33	As	34	Se	35	Br	36	Kr	39	Y	40	Zr
41	Nb	42	Mo	43	Tc	44	Ru	45	Rh	46	Pd	47	Ag	48	Cd
49	In	50	Sn	51	Sb	52	Te	53	I	54	Xe	55	Cs	56	Ba
57-70	lanthanoids														
71	Lu	72	Hf	73	Ta	74	W	75	Re	76	Os	77	Ir	78	Pt
79	Au	80	Hg	81	Tl	82	Pb	83	Bi	84	Po	85	At	86	Rn
87-102	actinoids														
103	Lr	104	Rf	105	Db	106	Sg	107	Bh	108	Hs	109	Mt	110	Uu
111	Uuh	112	Uuq	113	Uub	114	Uuq	115	Uub	116	Uuq	117	Uub	118	Uuo
89	Ac	90	Th	91	Pa	92	U	93	Np	94	Pu	95	Am	96	Cm
97	Bk	98	Cf	99	Es	100	Fm	101	Md	102	No	103	Lr	104	Rf

X-rays generally from fluorescence of scintillator or often from shielding.
Solution: move shielding far from detector and use a “graded shield”

Spectroscopy w/ Scintillators—Response Function

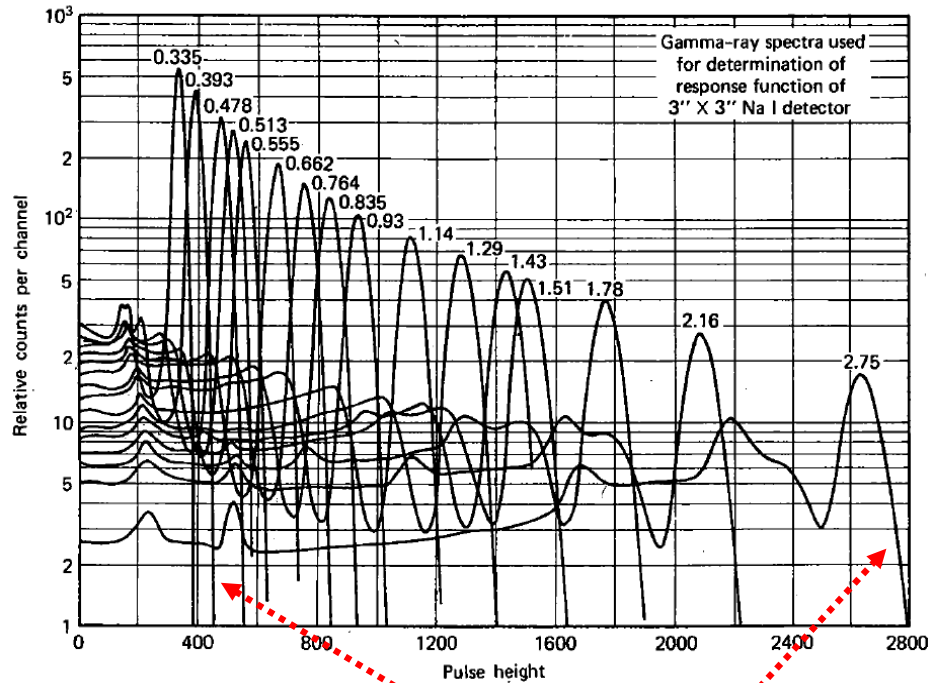


Fig. 10.11 & 22 Knoll, 3rd Ed.

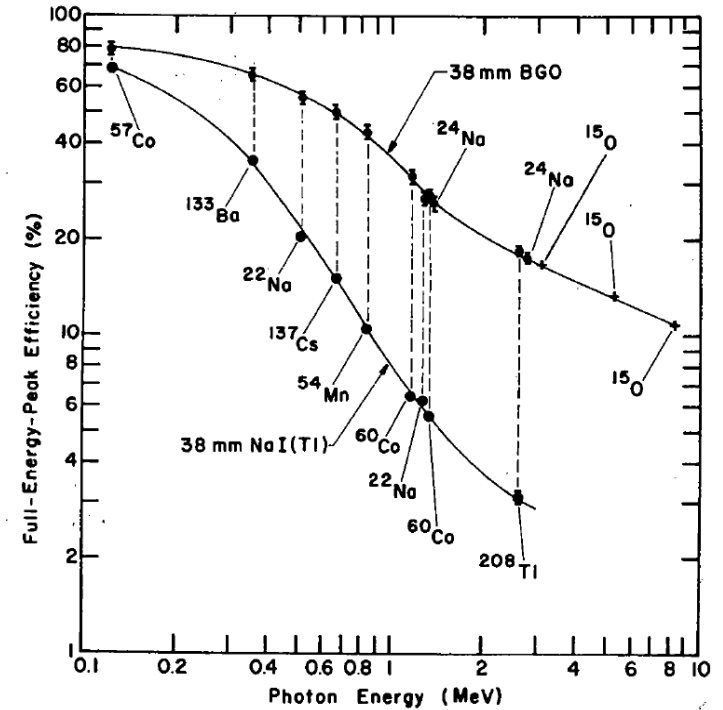
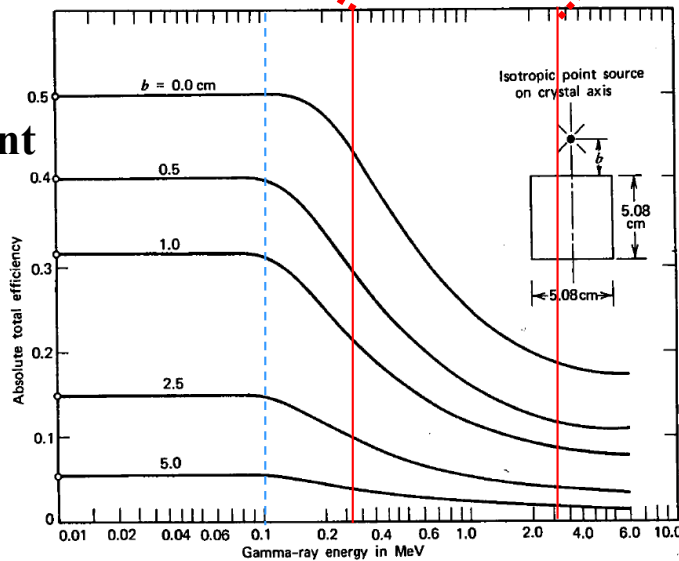


Fig. 10.28 Knoll, 3rd Ed.

$$\epsilon_{\text{total}} = \epsilon_{\text{geo}} \epsilon_{\text{int}}$$



Beware the crystal sizes are different in each of these figures!

Spectroscopy w/ Scintillators – Summing

Summing is a loss mechanism that depends on the geometry (true coincident) and on the rate (random).

True Coincident Summing: two photons are emitted in cascade from the same nucleus (within the total resolving time) and both strike the detector... ['A' is the activity of the source]

$$N_1 = \varepsilon_1^{Photo} \varepsilon_{Geo} (A * \Delta t) BR_1 \quad [= \varepsilon_1 \Omega S y_1 \quad \text{in text}]$$

$$N_2 = \varepsilon_2^{Photo} \varepsilon_{Geo} (A * \Delta t) BR_2$$

The sum peak:
$$N_{12}^{E-sum} = \left(\varepsilon_1^{Photo} \varepsilon_{Geo} BR_1 \right) \left(\varepsilon_2^{Photo} \varepsilon_{Geo} BR_2 \right) (A * \Delta t) W(\Theta = 0) \quad \propto \varepsilon_{Geo}^2$$

Any loss from "1":
$$N_{12}^{Loss} = \left(\varepsilon_1^{Photo} \varepsilon_{Geo} BR_1 \right) \left(\varepsilon_2^{Total} \varepsilon_{Geo} BR_2 \right) (A * \Delta t) W(0) \quad \propto \varepsilon_{Geo}^2$$

Observed "1":
$$N_1^{Net} = N_1 - N_{12}^{Loss} = N_1 \left[1 - \left(\varepsilon_2^{Total} \varepsilon_{Geo} BR_2 \right) W(0) \right]$$

Random Summing: two particles from different nuclei strike the detector within the total resolving time, τ , -- only depends on the total counting rate, r :

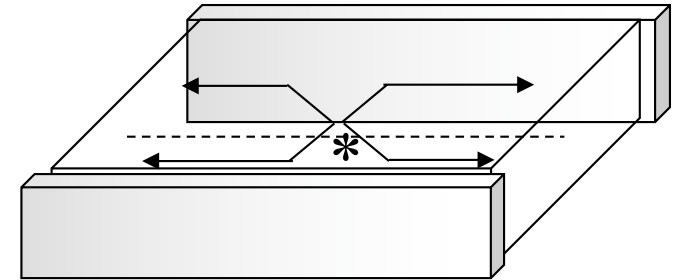
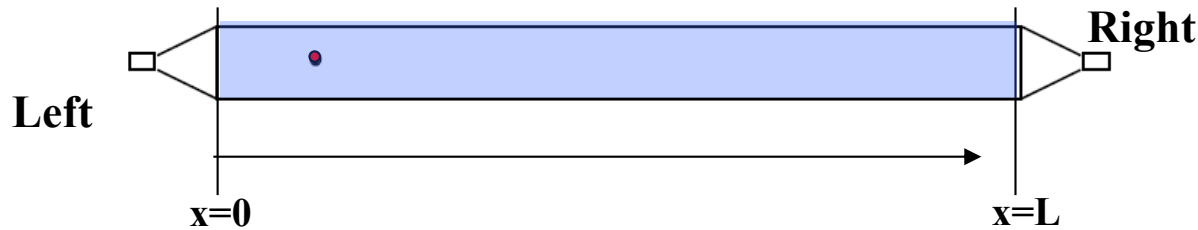
$$r_1 = \frac{N_1}{\Delta t} = A \left(\varepsilon_1^{Photo} \varepsilon_{Geo} BR_1 \right) \quad r_{12} = (r_1 \tau) r_{pu}$$

Where r_{pu} is the rate of all events that add signal onto "1" or pile up ... $r_{pu} = r_2 + r_1 + r_{back} + \dots$

N.B. random summing even occurs with sources that only emit one gamma ray!

Spectroscopy w/ Scintillators – Position

Position measurements can use the light output reaching each end of a bar or if the electronics are suitable, the time difference between signals at each end.



Assume that light is “piped” to the end with a Beer’s Law attenuation coefficient, α

$$I_L = \frac{I_0}{2} e^{-\alpha x} \quad I_R = \frac{I_0}{2} e^{-\alpha(L-x)}$$

$$I_L * I_R = \left(\frac{I_0}{2} e^{-\alpha x} \right) \left(\frac{I_0}{2} e^{-\alpha(L-x)} \right) = \left(\frac{I_0}{2} \right)^2 e^{-\alpha L} \quad \rightarrow \quad I_0 \propto \sqrt{I_L * I_R}$$

$$R = \frac{I_L}{I_R} = \frac{e^{-\alpha x}}{e^{-\alpha(L-x)}} = e^{+\alpha(L-2x)} \quad \rightarrow \quad \ln R \propto \frac{x}{L}$$

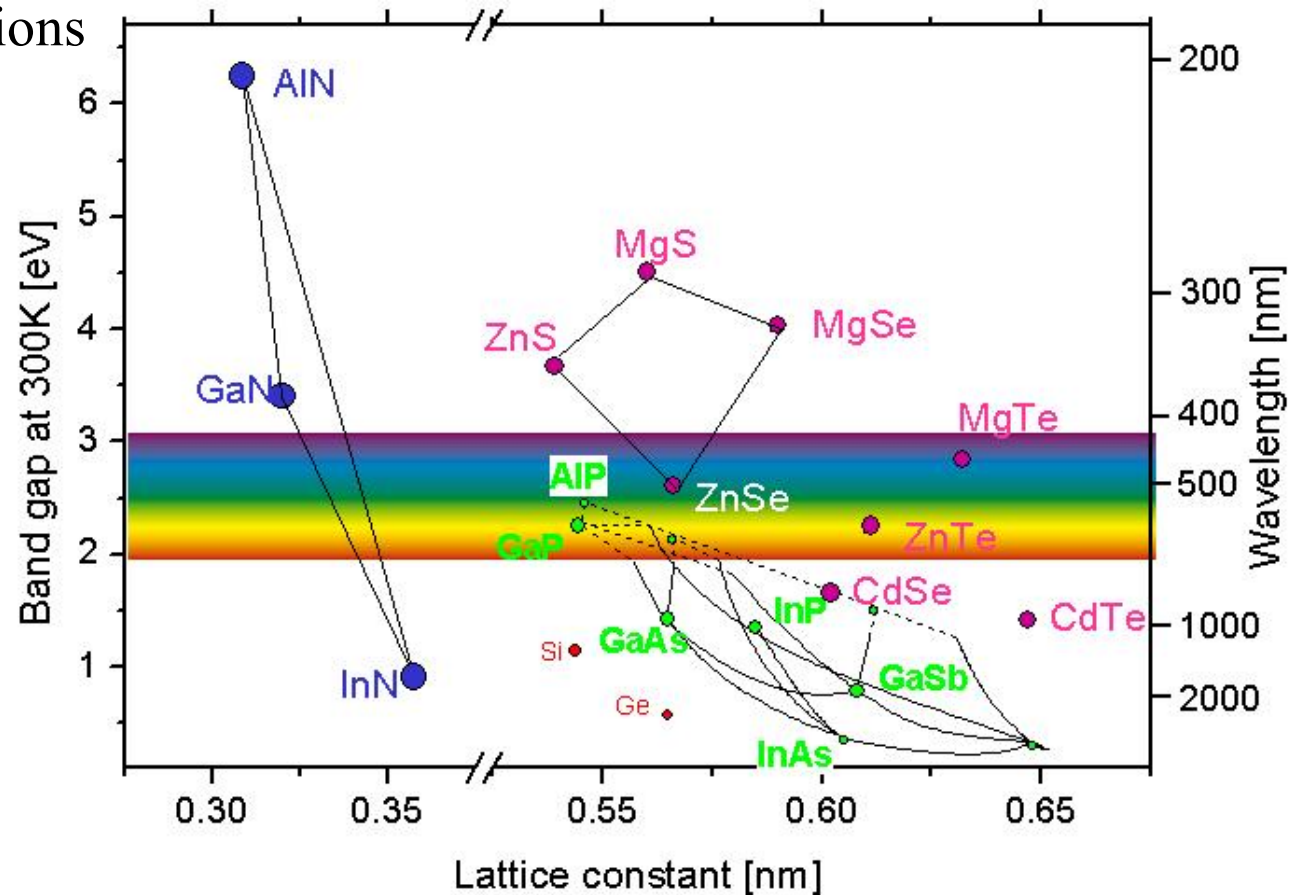
Week 7: Ch. 11 Semiconductor diodes

Principles of Scintillation Counters

Semiconductor Diodes

- basics of semiconductors
- pure elements & dopants
- 5-3 Materials
- ion collection, leakage current
- diode structure, pn, np junctions
- depletion region

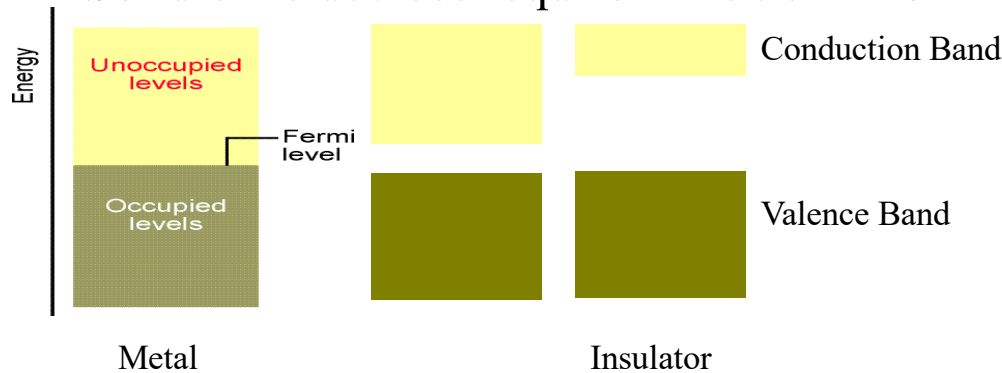
Practice Problems / Exam



Chap. 11 – Semiconductor Diodes

Semiconductor diodes provide the best resolution for energy measurements, silicon based devices are generally used for charged-particles, germanium for photons.

- Scintillators require ~ 100 eV / “information carrier” .. Photoelectrons in this case
- Gas counters require ~ 35 eV / “information carrier” .. Ion-pairs or Electrons
- Solid-state devices require ~ 3 eV / “information carrier” .. Electron/hole pairs



A semiconductor is an insulator with a small band gap, ~ 1.2 eV for silicon. Generally want smallest band gap *but* thermal excitation across the gap provides a leakage current. N.B. the actual band gap depends on the direction relative to the lattice (Si and Ge do not crystallize in cubic lattices) and the gap decreases slowly with temperature.

The ratio of ‘w’ to band gap is approximately constant for a wide range of materials – division of excitation energy between e/h pair and phonons, etc. is \sim constant.

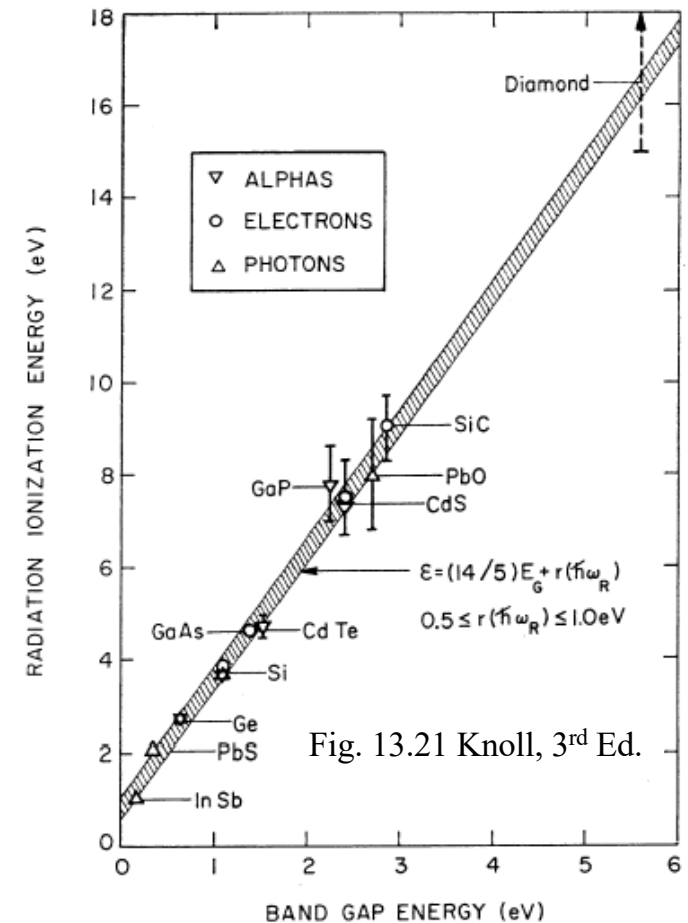


Fig. 13.21 Knoll, 3rd Ed.

Semiconductors – Charge carriers

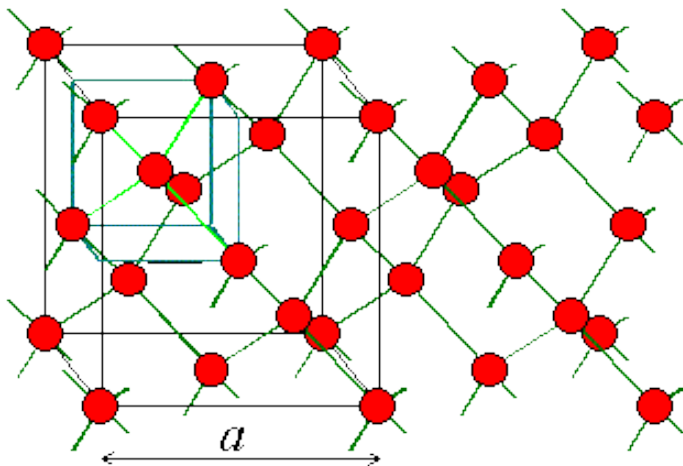
“pure” material, no dopants is called “intrinsic”

The intrinsic carrier density in a semiconductor is low:

$$\rho_e \sim \sqrt{N_V N_C} e^{-\varepsilon/2k_B T} \quad k_B T = 0.026 \text{ eV @ } 25^\circ \text{ C}$$

$$\rho_e \sim \sqrt{10^{19} 10^{19}} e^{-20} \sim 10^9 - 10^{10} \text{ cm}^{-3}$$

N_V and N_C are the densities of states in the valence and conduction bands. (Only rough estimates given here.)



“diamond lattice”

Lattice Constant

Carbon 0.356 nm
 Silicon 0.543 nm
 Germanium 0.565 nm

12.0111	-4
C	+2
6	+4
2-4	
28.0855	-4
Si	+2
14	+4
2-8-4	
72.59	-4
Ge	+2
32	+4
2-8-18-4	

Semiconductors – Dopants

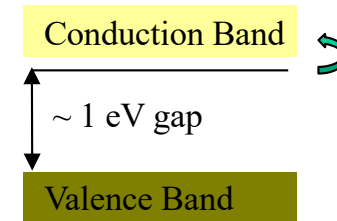
13	14	15
10.81 5 2-3 B +3	12.0111 6 2-4 C +2 +4	14.0067 7 2-5 N -3 -2 -1 +1 +2 +3 +4 +5
26.98154 13 2-8-3 Al +3	28.0855 14 2-8-4 Si +2 +4	30.97376 15 2-8-5 P -3 +3 +5
69.72 31 2-8-18-3 Ga +3	72.59 32 2-8-18-4 Ge +2 +4	74.9216 33 2-8-18-5 As -3 +3 +5

Add atoms from the neighboring groups in the periodic table

- Group 15, Phosphorous, nearly same size, excess electron
- Group 13, Boron, nearly same size, electron deficit

Donor level from P atom below conduction band by ~ 0.05 eV ..
Thermally excite from donor, excess electrons \rightarrow n-type

$$e^{-0.05/2kT} \sim e^{-1}$$



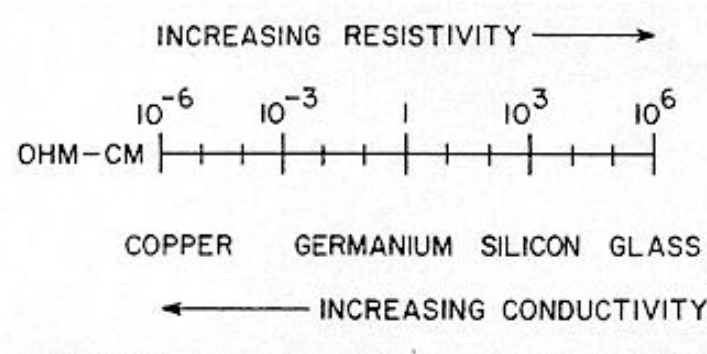
Conduction Band

~ 1 eV gap

Valence Band

Acceptor level from B atom above valence band by ~ 0.05 eV ..
Thermally excited from valence band, excess holes \rightarrow p-type

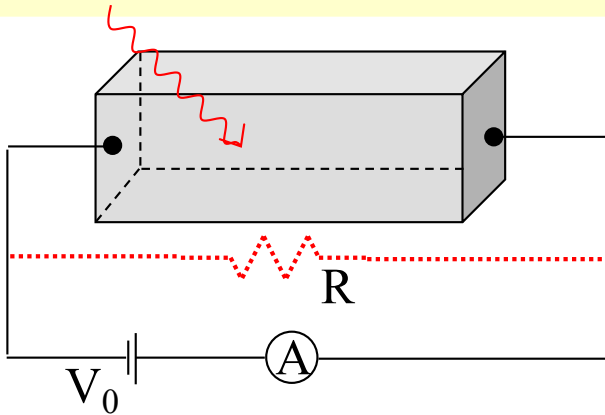
Control the conductivity by controlling the amount of dopants!
N.B. 2 ppb gives $(2 \times 10^{-9}) (5 \times 10^{22} / \text{cm}^3) = 10^{14} / \text{cm}^3 \gg 10^9$ for Si



For an n-type material, the electrons carry the current so that the resistivity is:

$$\rho = \frac{1}{q_e N_D \mu_e}$$

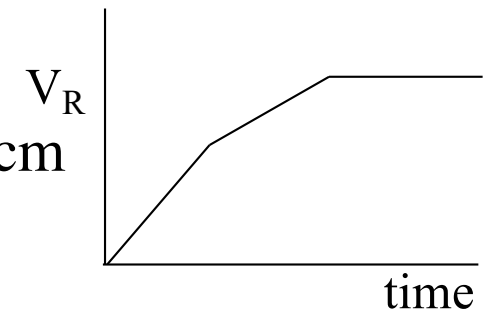
Semiconductor – Ion Chamber?



Imagine constructing a simple block of intrinsic semiconductor and trying to use it as an ion chamber ... The block has a length, “L” and a cross sectional area, “A” with a resistivity of $\rho = 60\text{k ohm-cm}$ (high quality silicon). Apply nominal $V_0 = 60\text{V}$ bias to collect ions.

1 MeV energy into material creates $\sim 3 \times 10^5$ e/h in ~ 50 ns ... limiting drift velocity $\sim 10^7$ cm/s

$$i_{\text{signal}} \sim \frac{\Delta q}{\Delta t} = \frac{3 \times 10^5 (1.6 \times 10^{-19})}{(L \text{ cm} / 10^7 \text{ cm} / \text{s})} = \frac{5}{L} 10^{-7} \text{ Amps for } L \text{ in cm}$$

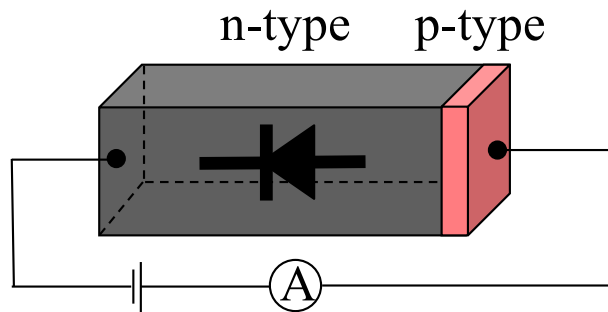


$$I_{\text{Leakage}} = \frac{V_0}{R} \quad \text{where} \quad R = \rho \frac{L}{A}$$

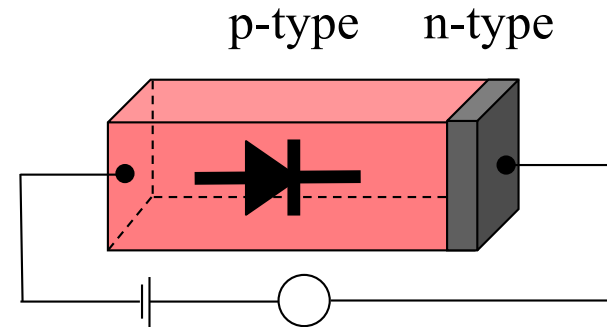
$$I_{\text{Leakage}} = \frac{V_0 A}{\rho L} \quad \rightarrow \quad \frac{60 A}{60,000 L} = \frac{A}{L} 10^{-3} \text{ Amps for } A/L \text{ in cm}$$

Thus, $I \gg i$ so we need a trick to kill the leakage current.

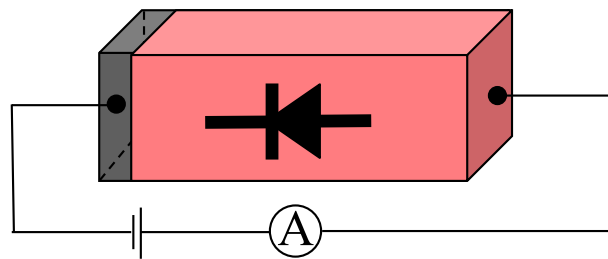
Semiconductor Diodes – 1



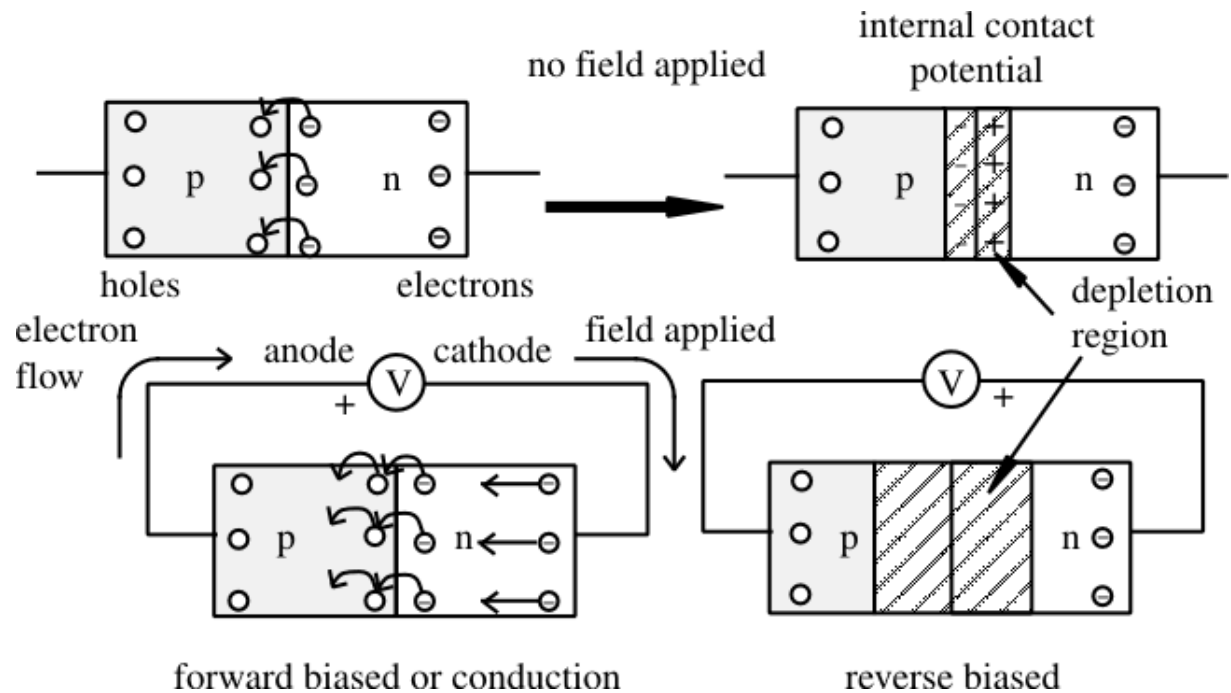
“Forward” bias – normal flow of current



“Reverse” bias – no nominal current



The “type” is determined by the implanted atoms .. Different atoms can be put into a single piece of semiconductor, then an internal field will form due to migration of charges.



Semiconductor Diodes – 2

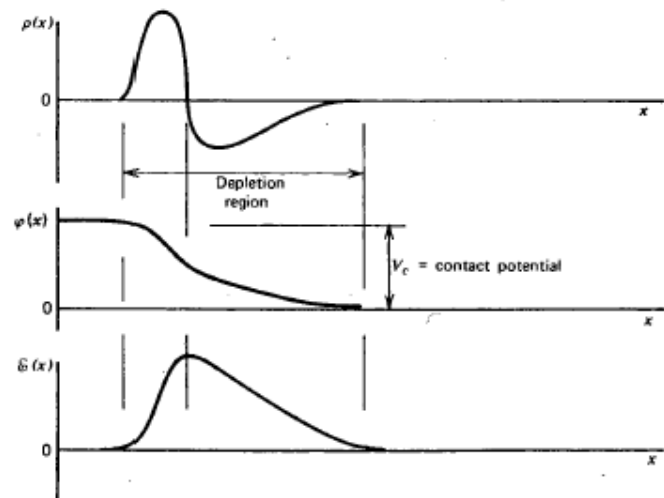
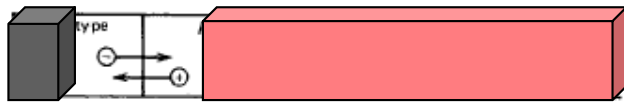
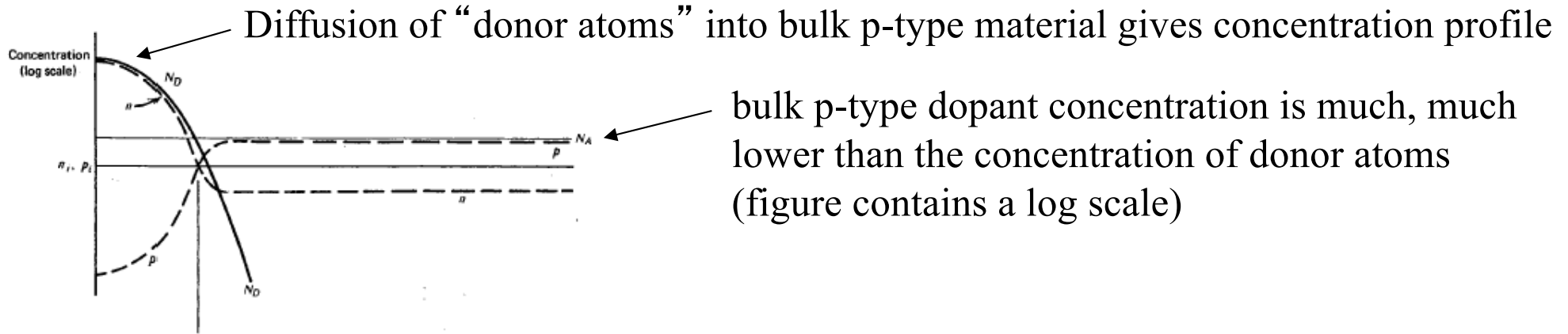


Fig. 11.8 Knoll, 3rd, 4th Eds.

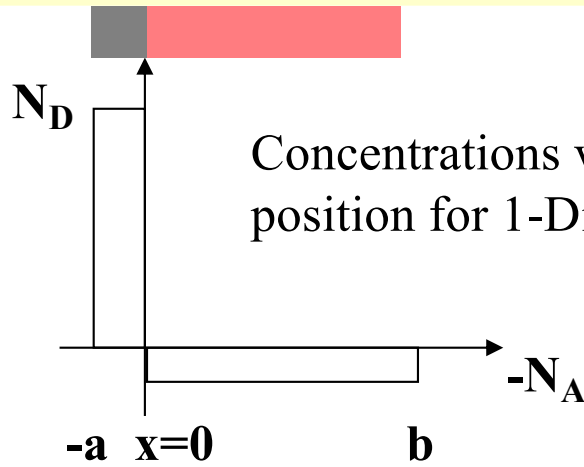
Charge density, $\rho(x)$: Migration of the charge across the boundary causes a charge separation and a “depletion region” ... in this case the holes migrate for a longer distance due to imbalance in concentrations in two regions

Which creates an internal electric potential, $\phi(x)$ with a potential difference of $\sim 1V$

and an electric field, $E(x)$, where the lines of force originate on positive ions and terminate on the negative ions.

The depleted region has a very low concentration of mobile charge carriers and a very high resistivity – this is a very good region to measure/collect ionization.

Semiconductor Diodes – Model for Depletion Depth



$$\text{Charge neutrality gives: } a \cdot N_D = b \cdot N_A \quad (1)$$

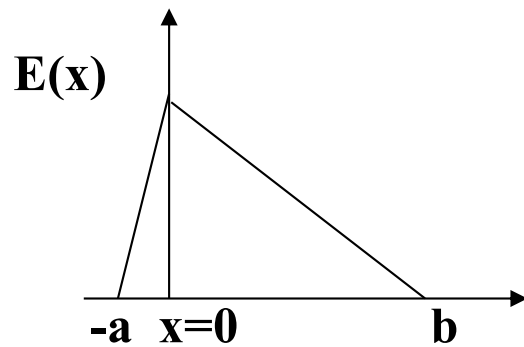
Depletion depth, $d = a + b$

One Dimensional Poisson Equation in two regions:

$$\frac{d^2\phi}{dx^2} = \frac{-\rho(x)}{\epsilon} = \begin{cases} \frac{-q_e N_D}{\epsilon} & a < x \leq 0 \\ \frac{+q_e N_A}{\epsilon} & 0 < x \leq b \end{cases}$$

Boundary Conditions :

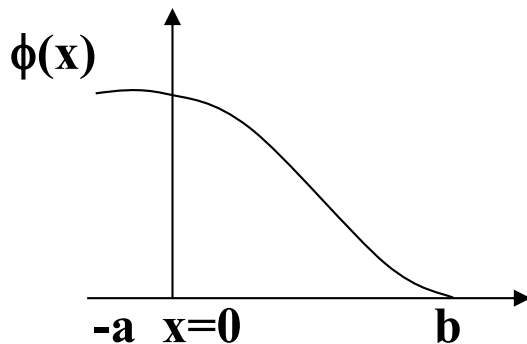
$$E(x = -a) = 0 \text{ \& } E(x = b) = 0$$



$$-E(x) = \frac{d\phi}{dx} = \begin{cases} \frac{-q_e N_D}{\epsilon} (x + a) & a < x \leq 0 \\ \frac{+q_e N_A}{\epsilon} (x - b) & 0 < x \leq b \end{cases}$$

Boundary Conditions :

$$\phi(x = -a) = V \text{ \& } \phi(x = b) = 0$$



$$\phi(x) = \begin{cases} \frac{-q_e N_D}{2\epsilon} (x + a)^2 + V & a < x \leq 0 \\ \frac{+q_e N_A}{2\epsilon} (x - b)^2 & 0 < x \leq b \end{cases}$$

Match at $x = 0$:

$$\frac{-q_e N_D}{2\epsilon} a^2 + V = \frac{+q_e N_A}{2\epsilon} b^2 \quad (2)$$

Depletion depth: combine (1) & (2) $(a+b) \cdot b = 2\epsilon V / q_e N_A$

Generally $b > a \quad \therefore d = b \sim (2\epsilon V / q_e N_A)^{1/2}$

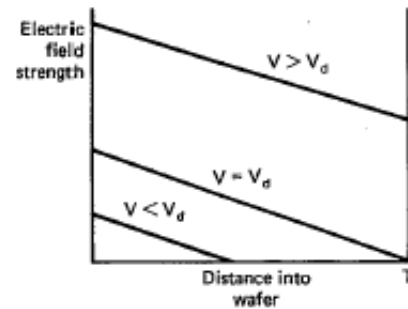
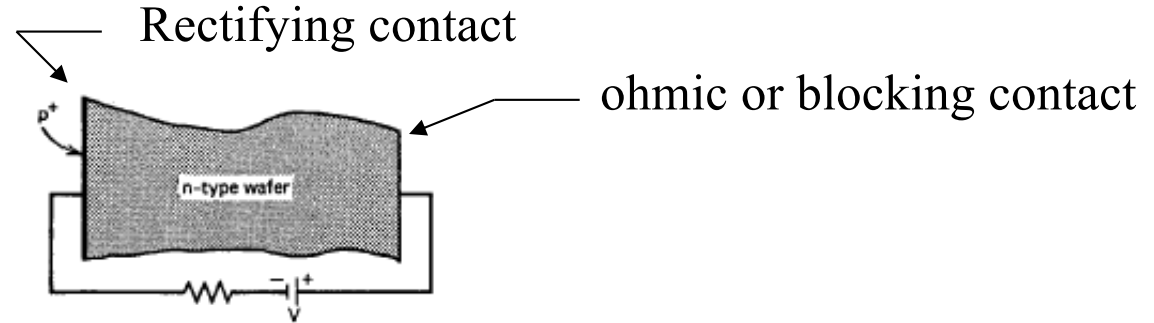
Semiconductor Diodes – 3

Some properties of the junction:

$$d \sim \sqrt{\frac{2\epsilon V}{q_e N_A}} \quad \& \quad \rho = \frac{1}{q_e N_D \mu_e}$$

$$d \sim \sqrt{2\epsilon V \rho \mu} \quad \& \quad C = \frac{\epsilon A}{d}$$

$$E = \Delta V / \Delta x \quad \rightarrow \quad E_{\max} = \frac{2V}{d}$$



One can apply an external (reverse) bias voltage such that the depletion layer extends from the junction to the full thickness of the device

Fig. 11.12 Knoll, 3rd, 4th Eds.

Silicon Surface Barrier Device

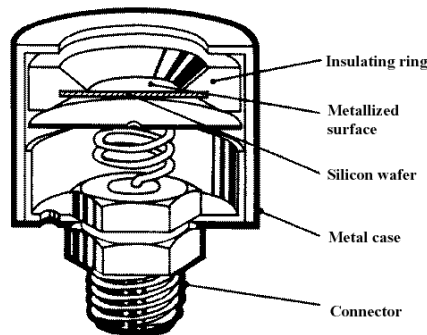
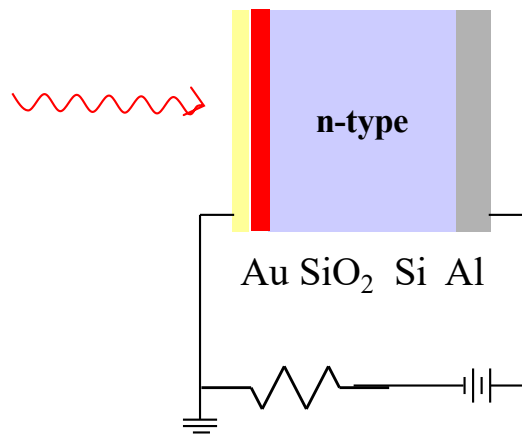


Fig. 11.11 Knoll, 3rd, 4th Eds. (From EG&G Ortec)

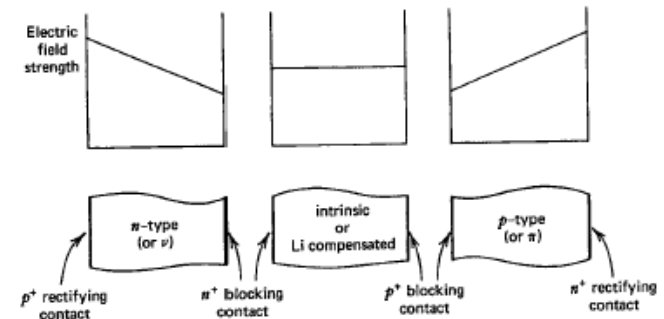
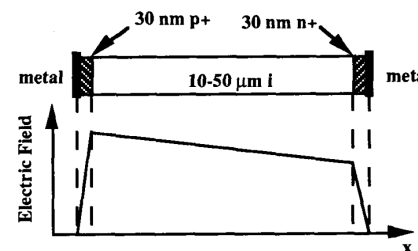
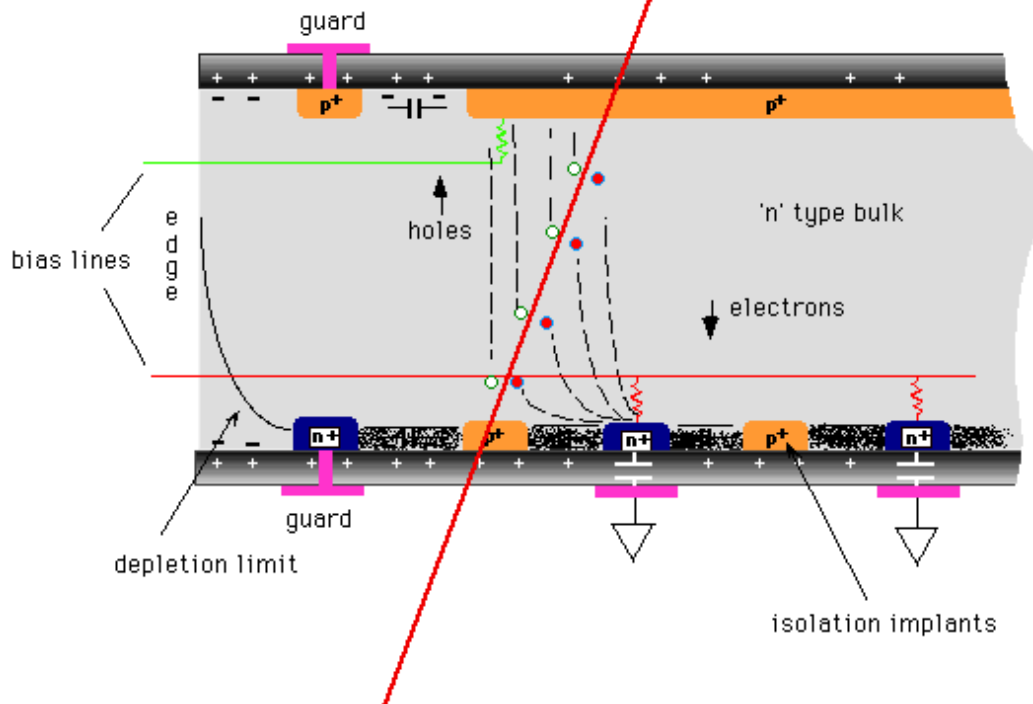


Fig. 11.13 Knoll, 3rd, 4th Eds.

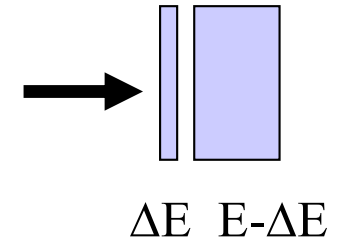


Semiconductor Diodes

ATLAS (HEP) 64x64x0.3 mm, n+ contacts 0.08 mm pitch



Silicon layers are thin, typically 0.3mm but up 5mm are produced. (dictated by the semiconductor chip industry).

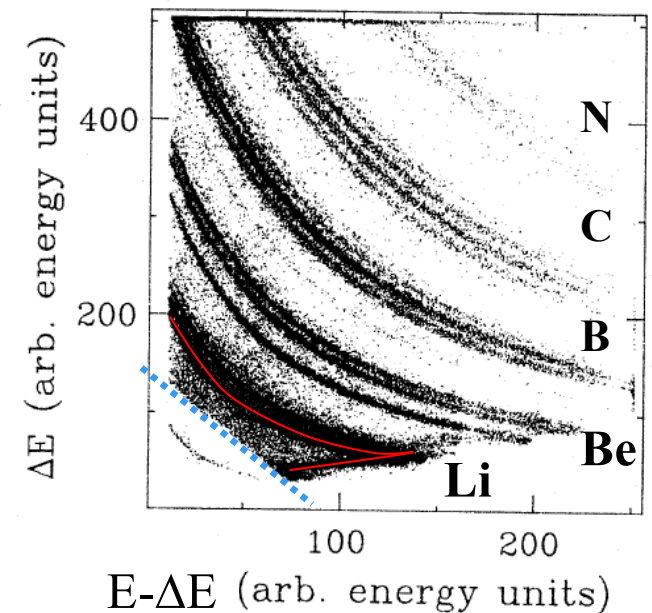


Silicon Detector “telescopes” combine a thin device with a thick device to identify charged particles.

$$\frac{dE}{dx} = C_1 \frac{MZ^2}{E} \ln\left(C_2 \frac{E}{M}\right)$$

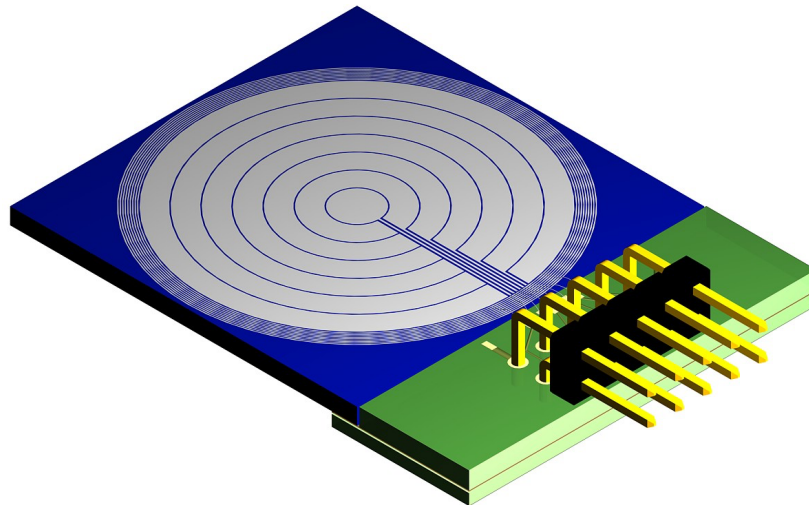
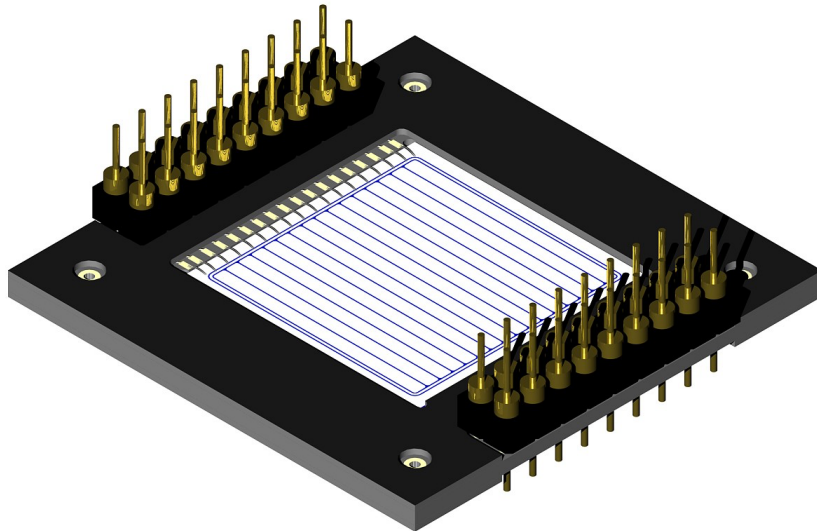
$$\Delta E = \Delta x \left(\frac{dE}{dx} \right) \propto \frac{MZ^2}{E} \rightarrow \Delta E \propto \frac{1}{E}$$

Punch-through
Software cut

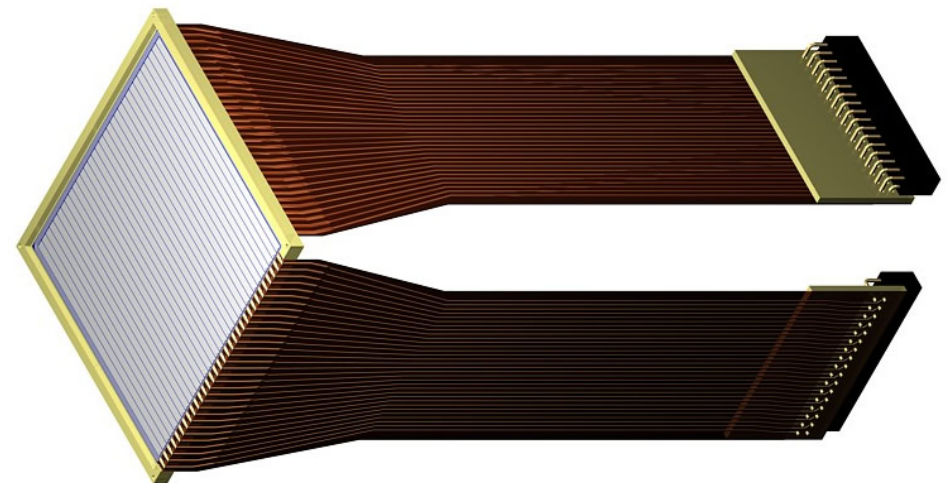


Semiconductor Diodes – Micron Semiconductor

<http://www.micronsemiconductor.co.uk>



WAFER SIZE	STANDARD SILICON THICKNESSES
	(μm)
4-inch	20, 30, 40, 50, 65, 80, 100, 140, 250, 300, 500, 1000, 1500
6-inch	150, 200, 300, 400, 500, 675, 1000



Week 8: Exam -1- Next Week

Semiconductor Diodes

Exam -1- in Classroom Tuesday, 17 Oct. 2017

-- Chapters 1 to 10 plus Vacuum

-- Open Book (Knoll)

Chemistry 985

Fall, 2011

Distributed: Tues., 18 Oct. 11, 8:30AM

Exam # 1 OPEN BOOK

Due: 18 Oct. 11, 10:00AM

1. (10 points)
the NSCL t
for neutrino
lar Mass=7
 ^{40}K ($T_{1/2}=1$
 ^{40}K to Ge ir

Chemistry 985

Fall, 2013

Distributed: Thurs., 17 Oct. 13, 8:30AM

Exam # 1 OPEN BOOK

Due: 17 Oct. 13, 10:00AM

2. Provide con
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the informat

Some constai

1. The textbc
a β ray (Q
single gam
and an x-r
and a differ
all covered
to absorb s

Fall, 2015

Distributed: Mon., 19 Oct. 15, 12:30PM

Chemistry 985

Exam # 1 OPEN BOOK

Due: 19 Oct. 15, 1:45PM

- (a) (10 poi
tube?

Some co

- (a) (2 points) vacuum air

Chemistry 985

Fall, 2017

Distributed: Mon., 17 Oct. 17, 8:30AM

Exam # 1 OPEN BOOK

Due: 17 Oct. 17, 10:00AM