Derivation of Nernst Equation

\[ \Delta G = \Delta G^0 + RT \ln Q \quad Q = \text{reaction quotient} \]

\[ Q = \frac{[\text{Products}]}{[\text{Reactants}]}, \]

\[ O^x + n e^- \rightarrow \text{RED} \]

1. \( \Delta G = -nFE \) \hspace{1cm} \text{non-standard condition}
2. \( \Delta G^0 = -nFE \) \hspace{1cm} \text{standard condition}

2. Substituting into the 1st equation gives:

\[ -nFE = -nFE^0 + RT \ln Q \]

3. Divide through by \(-nF\) gives:

\[ E = E^0 - \frac{RT \ln Q}{nF} \]

\[ \text{Subr.} \]

\[ E = E^0 - \frac{RT \ln \frac{[\text{RED}]}{[\text{PROP}]}}{nF} \]

4. Assume 25°C and converting \( \ln u \) to \( \log x \)

\[ \ln x = \ln (10^{0.301}) = \log x \left( \log 10 \right) \rightarrow 2.303 \]

\[ E = E^0 - \frac{0.383RT \log \frac{[\text{RED}]}{[\text{PROP}]}}{nF} \quad \text{at 25°C} = 0.0591 \]

\[ E = E^0 - \frac{0.0591 \log \frac{[\text{RED}]}{[\text{PROP}]}}{n} \quad \text{or} \quad E = E^0 + \frac{0.0591 \log \frac{[\text{RED}]}{[\text{PROP}]}}{n} \]
Collapse of BV Equation into a Thermodynamic Form

\[ i = \frac{nF}{RT} \left[ C_0(t) \exp(-nxFn) - C_p(t) \exp((1-x)NFn) \right] \]

1. At equilibrium, \( i = 0 \) so \( \frac{iF}{nF} = \frac{C_0(t)}{C_p(t)} \) exchange rates are equal
\[ \frac{nF}{RT} C_0(t) \exp(-nxFn) = nF \frac{C_0(t)}{C_p(t)} \exp((1-x)NFn) \]

2. Divide both sides by \( nF \), this leaves:
\[ C_0(t) \exp(-nxFn) = C_p(t) \exp((1-x)NFn) \]

3. Expand right exponential:
\[ C_0(t) \exp(-nxFn) = C_p(t) \exp(nF \frac{E - E^0}{RT}) \exp(-nxFn) \]

4. Divide both sides by \( C_p(t) \), this gives:
\[ \frac{C_0(t)}{C_p(t)} \exp(-nxFn) = \exp(nF \frac{E - E^0}{RT}) \exp(-nxFn) \]

5. Divide both sides by \( \exp(-nxFn) \), this gives:
\[ \frac{C_0(t)}{C_p(t)} = \exp(nF \frac{E - E^0}{RT}) \]

6. Take log of both sides, this gives:
\[ \ln \frac{C_0(t)}{C_p(t)} = nF \frac{E - E^0}{RT} \]

7. Multiply both sides of equation by \( \frac{RT}{nF} \), this gives:
\[ \frac{RT}{nF} \ln \frac{C_0(t)}{C_p(t)} = E - E^0 \]
}[add \( +E^0 \) to both sides]
\[ E^0 + \frac{RT}{nF} \ln \frac{C_0(t)}{C_p(t)} = E \]

Collapse to Nernst equation!