Week 7 Lecture 3 – Nuclear Reactions

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Nuclear Reactions

-- Nuclear Reactions & Waste
-- Transmutation
-- Nuclear Reactions overview
--- neutron induced reactions
--- charged particle induced reactions
-- Cross Section, Target attenuation
-- Energetics
-- Conservation of Momentum

No Homework this week,
Practice Exam on Monday

Cross Sections & Beam Attenuation

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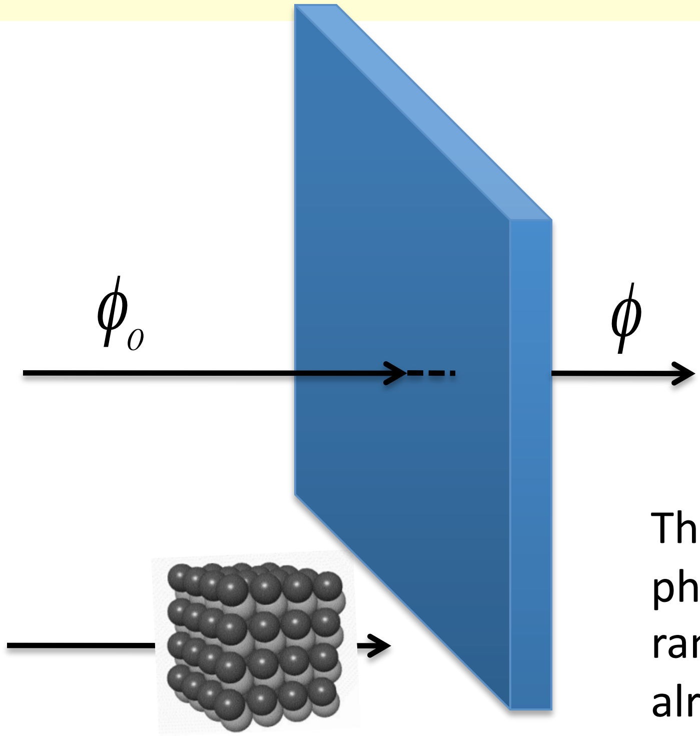


$$\frac{d\phi}{dx} = -\phi\sigma\rho_n$$

$$\frac{d\phi}{\phi} = -\sigma\rho_n dx \quad \rightarrow \quad \ln\frac{\phi}{\phi_o} = -\sigma\rho_n x$$

$$\rightarrow \quad \phi = \phi_o e^{-\sigma\rho_n x}$$

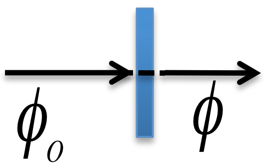
The exponential attenuation of a beam is a very general phenomenon that occurs whenever the beam interacts randomly with a uniform medium. You might have already encountered this behavior as the "Beer-Lambert Law" for the absorption of light.

Macroscopic scale: all neutrons and most energetic ions will penetrate thin metal foils… the nuclei take up such a small volume we can imagine that the foil is only two dimensional. $\rho_n\, x = \rho_A$ "areal density" This concept solves the practical problem of trying to measure the thickness of these foils. So the attenuation of the beam is thus:

$$\ln\frac{\phi}{\phi_o} = -\sigma\rho_A \quad \rightarrow \quad \phi = \phi_o e^{-\sigma\rho_A}$$

Beam Attenuation & Reaction Rate

The number of products of the reaction is simply calculated from the missing beam …



$$N_{rxn} = \left(\phi_o - \phi\right) \quad but \quad \phi = \phi_o e^{-\rho_A \sigma}$$

$$N_{rxn} = \left(\phi_o - \phi_o e^{-\rho_A \sigma}\right) \quad but \quad e^{-x} \approx 1 - x + \frac{x^2}{2} - \quad when\ x < 1$$

$$N_{rxn} = \rho_A \sigma \phi_o$$

The beam could be quoted as total number of particles (fluence) or the particle rate (flux). The flux is usually more useful for the production of radioactivities.

Reaction Threshold Energy

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The kinetic energy of the reactants can often make a substantial contribution to the total energy of a nuclear reaction, for example, MeV's of kinetic energy compared to Q-values on the same order. This is again in contrast to chemical reactions where the typical kinetic energy is on the order of k_BT ~ 0.025eV and chemical bond energies are orders of magnitude larger, ~ eV's.

$$^{4}He^{2+} + ^{14}N \rightarrow \left(^{18}_{9}F_{9}^{2+}\right)^{*} \rightarrow ^{17}O^{+} + ^{1}p^{+} + Q$$

$$Q = \left[\Delta(^{4}He) + \Delta(^{14}N)\right] - \left[\Delta(^{17}O) + \Delta(^{1}H)\right]$$

$$Q = -1.192 MeV$$

This reaction has a negative Q-value and is endoergic… the reaction requires energy to go in the forward direction. This is called the *threshold energy* for the reaction. Rutherford was able to carry out this reaction using alpha particles from a source with $T_\alpha > |Q|$

This reaction can be written in a shorthand notation: $^{14}N(\alpha,p)^{17}O$ or even just $^{14}N(\alpha,p)$ because we know that nuclear reactions have to balance with respect to baryon number, lepton number, linear momentum, angular momentum, and total energy!! We discussed the balancing of nuclear reactions in the framework of nuclear decay, a subset of reactions.

The format is: target(projectile, projectile-like product)target-like product
The target-like product can be omitted in two-body reactions, just like we omit the neutron number and chemists can omit the atomic number.

$^{208}Pb(^{12}C,\alpha)$ -> ^{12}C + ^{208}Pb -> ^{4}He + ^{216}Rn

Conservation of Energy and Momentum

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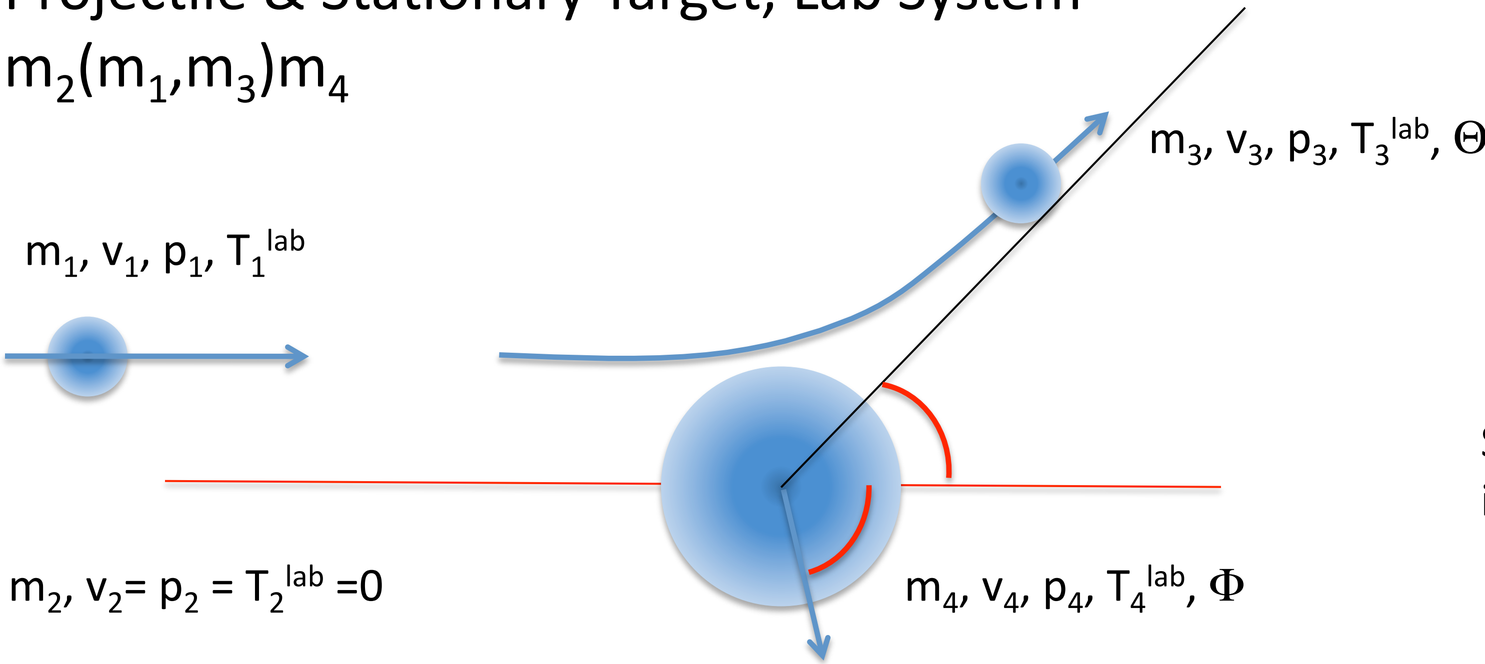
Projectile & Stationary Target, Lab System

$m_2(m_1,m_3)m_4$



Similar to Fig. 10-1 in the text

Conservation of (linear) momentum: $m_1v_1 + 0 = m_3v_3 \, Cos\Theta + m_4v_4 \, Cos\Phi$

$0 + 0 = m_3v_3 \, Sin\Theta - m_4v_4 \, Sin\Phi$

Conservation of Energy: $T_1^{lab} + 0 = T_3^{lab} + T_4^{lab} + Q$

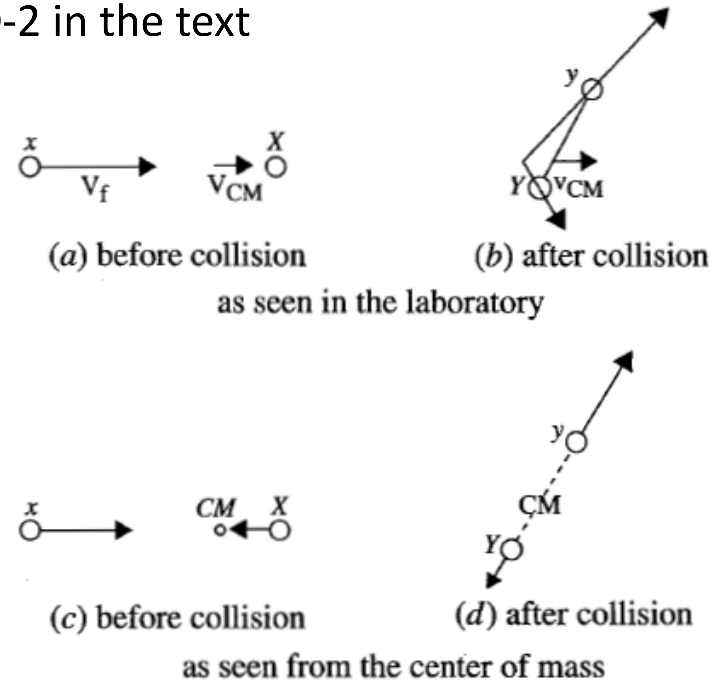
or $p_1^2/2m_1 = p_3^2/2m_3 + p_4^2/2m_4 + Q$

This set of simultaneous equations can be solved for Q in terms of the angle Θ which provides a scattering technique to measure Q in the lab and determine masses of exotic nuclei.

Center of Mass System

Notice that these nuclear collisions have the feature that some of the initial kinetic is converted into recoil energy and is not available, so to speak, to be used in the reaction. This energy is associated with the motion of the “center of mass” of the collision partners which must be the same before and after the collision. $T_{CMS} = T_1^{lab} (m_1/ m_1+m_2)$

Fig. 10-2 in the text



A mathematical construction that builds in this correction automatically and that also simplifies the interpretation of the collision is to consider the reaction in the center of mass reference frame. The CMS system has the feature that the momenta are equal and opposite in the X and Y coordinates both before and after the collision.

Going back to the threshold for a endoergic nuclear reaction and including the correction for the center of mass motion: Threshold Projectile (lab) $T = |Q| (m_1+m_2)/m_2$

Capture Reactions

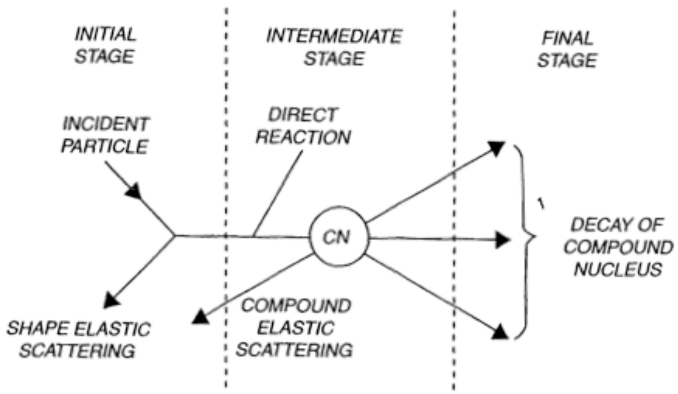
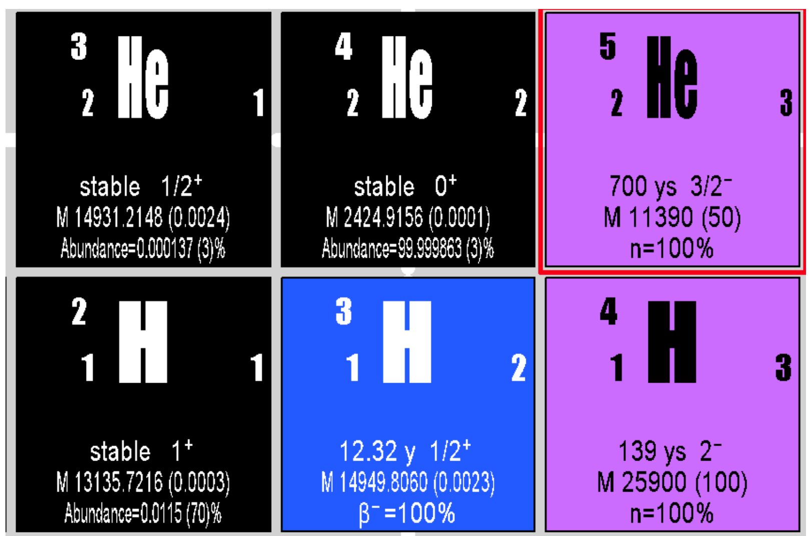


Fig. 10-3
in the text

"D-T" $^{3}t + {}^{2}d \rightarrow ({}^{5}He)^{*} \rightarrow {}^{4}He + {}^{1}n$



Fusion Reactions