Week 6 Lecture 2 – Internal Transitions

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Gamma Radiation

-- Industrial Applications of Radiation

-- Internal Transitions revisited

-- Nuclear Energy Levels

5th Homework
due Monday

Cell phone antenna …

ν = 850 MHz .. $\lambda = c/\nu = 0.35$m .. quarter wave

Odd particle angular momentum coupling

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$$ {}^{60}_{27}Co_{33} \quad \pi(f^{-1}_{7/2}) \quad \upsilon(f^{1}_{5/2}) $$

This is Hole & Particle configuration:
$j_1 = 7/2$ ($j_1 = l_1$ +S) $j_2 = 5/2$ ($j_2 = l_2$ -S)

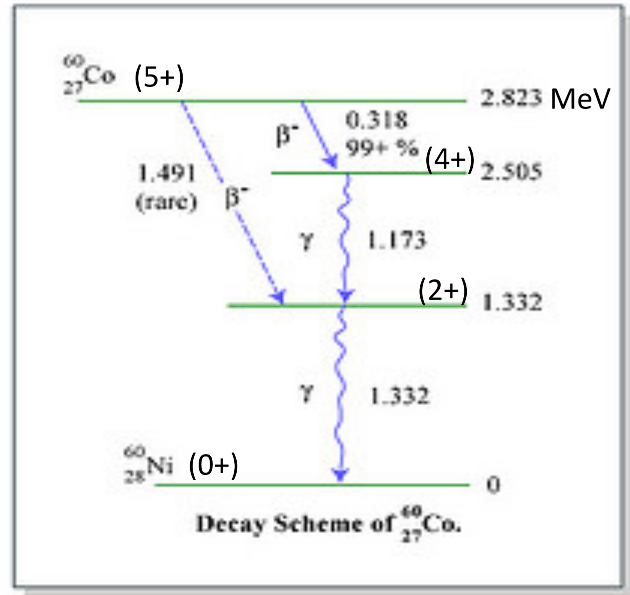
Brennan – Bernstein Rules (p.146 in text)

<u>Two particles or Two holes:</u>
R1: $j_1 = l_1$ +/- S & $j_2 = l_2$ -/+ S
$I = | j_1 - j_2 |$

R2: $j_1 = l_1$ +/- S & $j_2 = l_2$ +/- S
$I = | j_1 +/- j_2 |$

<u>One particle and one hole:</u>
R3: $I = j_1 + j_2 - 1$

Decay process for ^{60}Co:



Beta decay from I=5 state can occur by an allowed transition, Δ L = 0, with Δ S = 1 to the I=4 state. Beta decay to the lower states require Δ L > 0 and take place much more slowly (thus are labeled as "rare" in this diagram).

The decay from the I=4 excited state in the daughter nucleus can (only) take place by photon emission.
Two choices: I=4 to I=2 then I=2 to I=0 ground state
or I=4 to I=0 ground state.
Q: What does nature prefer?

Photons and Angular Momentum

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Facts about photons (you might not have ever known):

•Photons carry integral units of angular momentum, 1h-bar , 2 h-bar, etc.
the radiation pattern will reflect the angular momentum of the photon

•Photons are characterized as either Electric polarization or Magnetic polarization that is created by the motion of charge at the source (antenna, molecule, atom or nucleus).

•Photons have a parity (reflection symmetry) that depends on their L value and character:
Electric has $\pi = (-1)^L$ Magnetic has $\pi = (-1)^{(L-1)}$

Radiation	Δ L	$\Delta \pi$	Name
E1	1	yes	Electric Dipole
M1	1	no	Magnetic Dipole
E2	2	no	Electric Quadrupole
M2	2	yes	Magnetic Quadrupole

See Table 9-1 in the text, up to 4

Note that changes in atomic and molecular systems are almost always associated with changes in the distribution of electric charge and are mediated by “dipole” photons (E1).

Changes inside a nucleus can be associated with a change in the distribution of electric charge (proton changes orbital) and thus “electric” transitions E1, E2, etc.
But changes in the electrical current are also possible (change in the orientation of a proton orbital) leads to “magnetic” transitions M1, M2, etc.

Example of Photon Selection Rules

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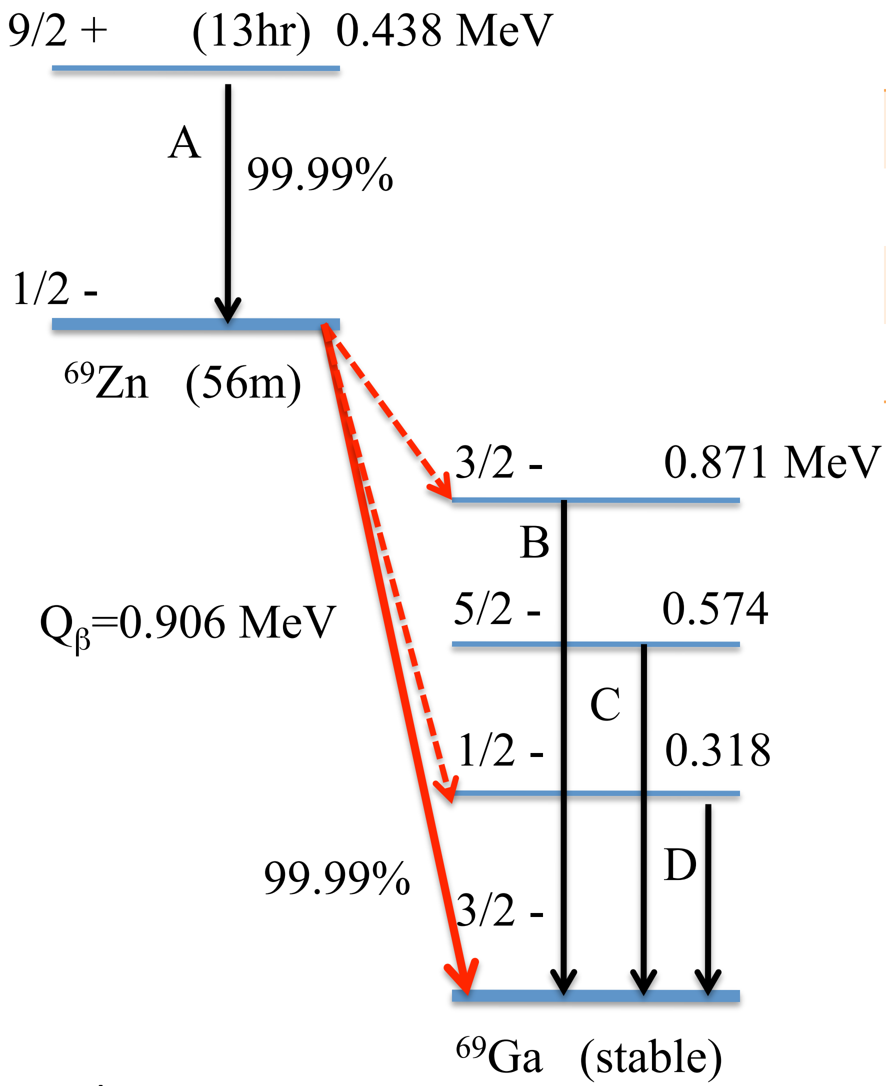
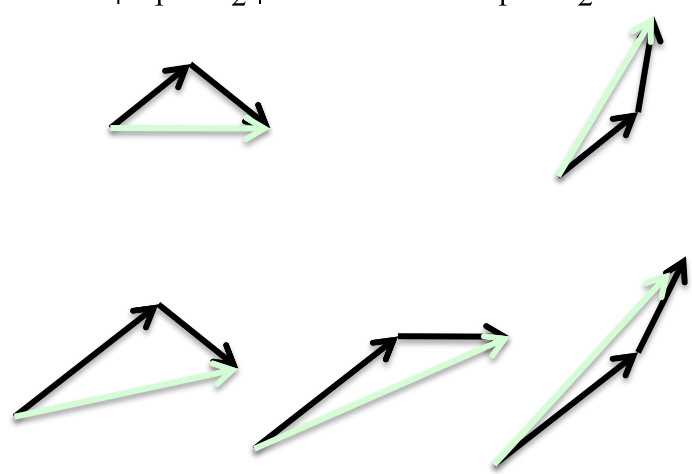


Fig. 9-1 in the text

Photon	Δ L	Δ π	Lowest Type
A	4, 5	yes	M4
B	0,1,2,3	no	(no E0) M1
C	1,2,3,4	no	M1
D	1,2	no	M1

Angular momentum coupling

$$|I_1 - I_2| \ldots \ \Delta L \ \ldots I_1 + I_2$$



Photon decay rates – 1

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The general form of the decay constant for transitions between nuclear states relies again on work by Enrico Fermi and is outlined in the text. The expression is slightly more complicated than that for beta decay because we have to consider the multipolarity and character of the transition. The expression also depends on an overlap integral that contains the QM initial and final states (but written in a slightly different format than beta decay).

$$\lambda\left(l, I_i, \pi_i \rightarrow I_f, \pi_f\right) = (\text{constants}) \frac{(l+1)}{l[(2l+1)!!]^2} (E_\gamma)^{2l+1} B(l, I_i, \pi_i \rightarrow I_f, \pi_f)$$

The overlap integral has to be evaluated in each case (too big a job) so simplified expressions have been developed assuming the transitions occur by the motion of a single particle inside a uniformly distributed nucleus. These expressions then only rely on the gamma ray and the nuclear size:

$$B_{SP}(E,l) = \frac{1}{4\pi}\left[\frac{3}{l+3}\right]^2 (r_0)^{2l} A^{2l/3}$$

$$B_{SP}(M,l) = \frac{10}{\pi}\left[\frac{3}{l+3}\right]^2 (r_0)^{(2l-2)/2}$$

For example, substituting for *l=1:*

$$\lambda_{SP}(E1) = \frac{8\pi(2)}{1[3!!]^2} \left(\frac{E_\gamma}{\hbar c}\right)^3 \left[\frac{3}{4}\right]^2 (1.2\, fm)^2 A^{2/3} \frac{e^2}{4\pi\hbar} \rightarrow \quad = 1.03x10^{14} E_\gamma^3 A^{2/3} \quad / s$$

Photon decay rates – 2

The general result for the single particle transition rates made by Weisskopf is that the lowest multipolarities occur the fastest …

Table 9-2 in the text

L	Electric λ (s^{-1})	Magnetic λ (s^{-1})
1	$1.03\times10^{14}\ A^{2/3}\ E_\gamma^{\ 3}$	$3.15\times10^{13}\ E_\gamma^{\ 3}$
2	$7.28\times10^{7}\ A^{4/3}\ E_\gamma^{\ 5}$	$2.24\times10^{7}\ A^{2/3}\ E_\gamma^{\ 5}$
3	$3.39\times10^{1}\ A^{2}\ E_\gamma^{\ 7}$	$1.04\times10^{1}\ A^{4/3}\ E_\gamma^{\ 7}$
4	$1.07\times10^{-5}\ A^{8/3}\ E_\gamma^{\ 9}$	$3.27\times10^{-6}\ A^{2}\ E_\gamma^{\ 9}$

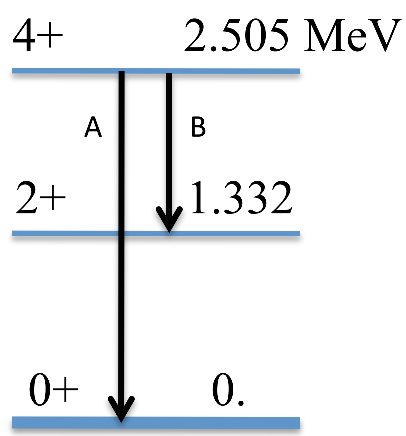
A) $\Delta L=4$, $\Delta\pi$=no requires E4 with E_γ= 2.505 MeV

B) $\Delta L=2$, $\Delta\pi$=no requires E2 with E_γ = 1.173 MeV

$$\frac{\lambda(E4)}{\lambda(E2)} = \frac{1.07x10^{-5}A^{8/3}(2.505)^9}{7.28x10^{7}A^{4/3}(1.173)^5}$$

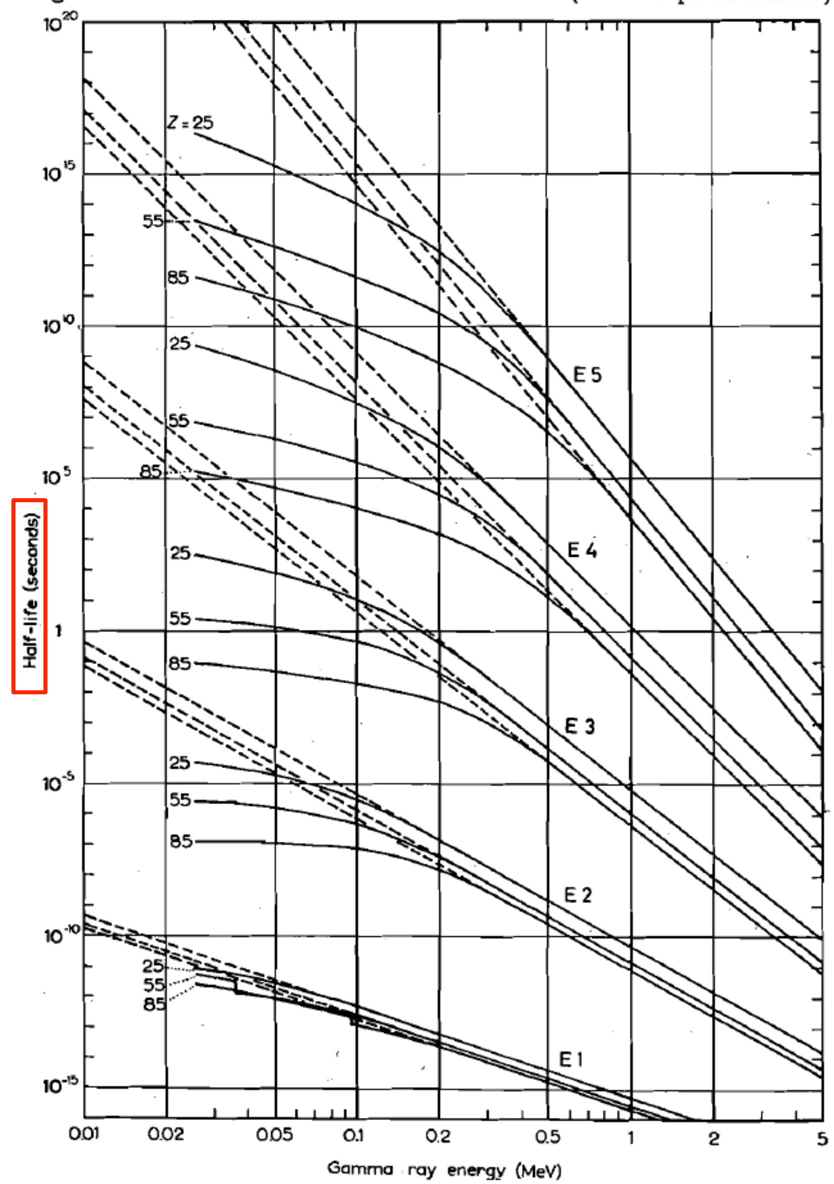
$$\frac{\lambda(E4)}{\lambda(E2)} = 1.47x10^{-13}A^{4/3}(1749)$$

$$\frac{\lambda(E4)}{\lambda(E2)} = 6.04x10^{-8}$$



^{60}Ni (stable)

Fig. 8a. Half–lives of electric transitions (Weisskopf estimate)



Inverse of Fig. 9-3 in the text