Week 4 Lecture 2 – Basics of Nuclear Structure

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Basics of Nuclear Structure

-- Nuclear sizes & shape

-- Nuclear force viz. Coulomb potential

-- Nuclear potential well

-- Schematic shell model of nuclei

--- Predictions of shell model

3rd Homework due Monday

Arc de Triumphe, Interior

Potential Energy Function for Nucleus

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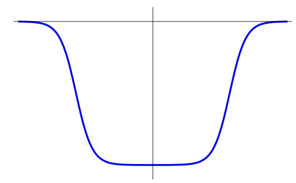
The overall result is a convention to treat the neutrons and protons as if they sit in separate potential energy wells in the nucleus. The mathematical form for the nuclear part of the interaction is called the Woods-Saxon form and is given by the expression:

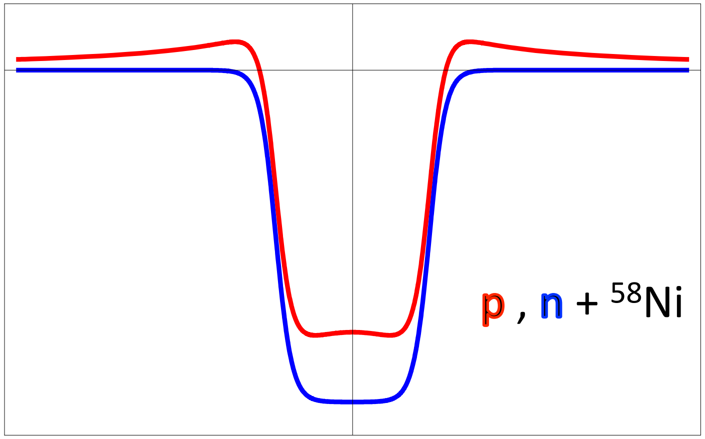
$$V_{nuclear}(r)[V, R_{sa}, a] = \frac{V}{1 + e^{-|r - Rsa|/a}}$$



Where V is the strength, R_{sa} is the “nuclear radius” and *a* is the diffuseness and all are parameters to be evaluated for a given nucleus.

This function provides the shape of the quantum mechanical potential energy for the neutrons but it must be combined with the coulomb potential to give the total potential energy “felt” by the protons.

Features of the separate P.E.’s:
1) The two wells will be similar for light nuclei with small Coulomb interactions.
2) The well will be deeper for the neutrons than for the protons in heavy nuclei.



Aside: Angular Momentum

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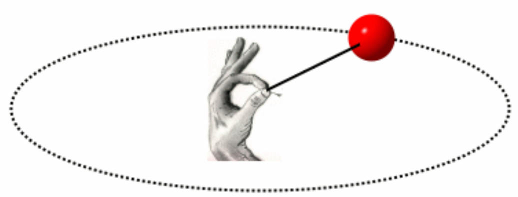
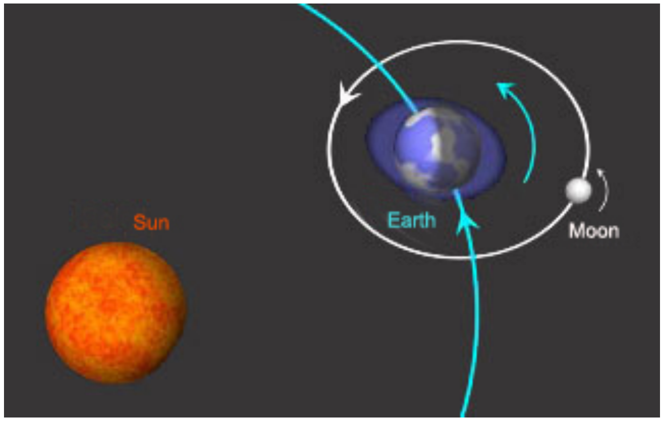
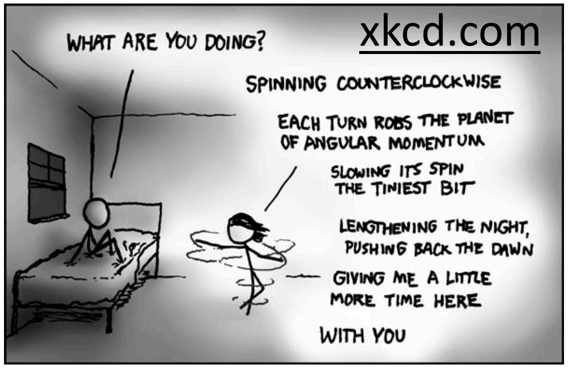
The angular momentum carried by a particle in a quantum mechanical system falls into two general classes:

(1) orbital motion, which is in some sense "optional" for the particle and depends on the detailed motion



(2) intrinsic spin, which is a characteristic of the particle itself and cannot be changed.





Note that both of these motions can be present at the same time and lead to a combined overall motion. The energy of the state depends, as usual, on the total.

Conceptual Energy Level Diagram

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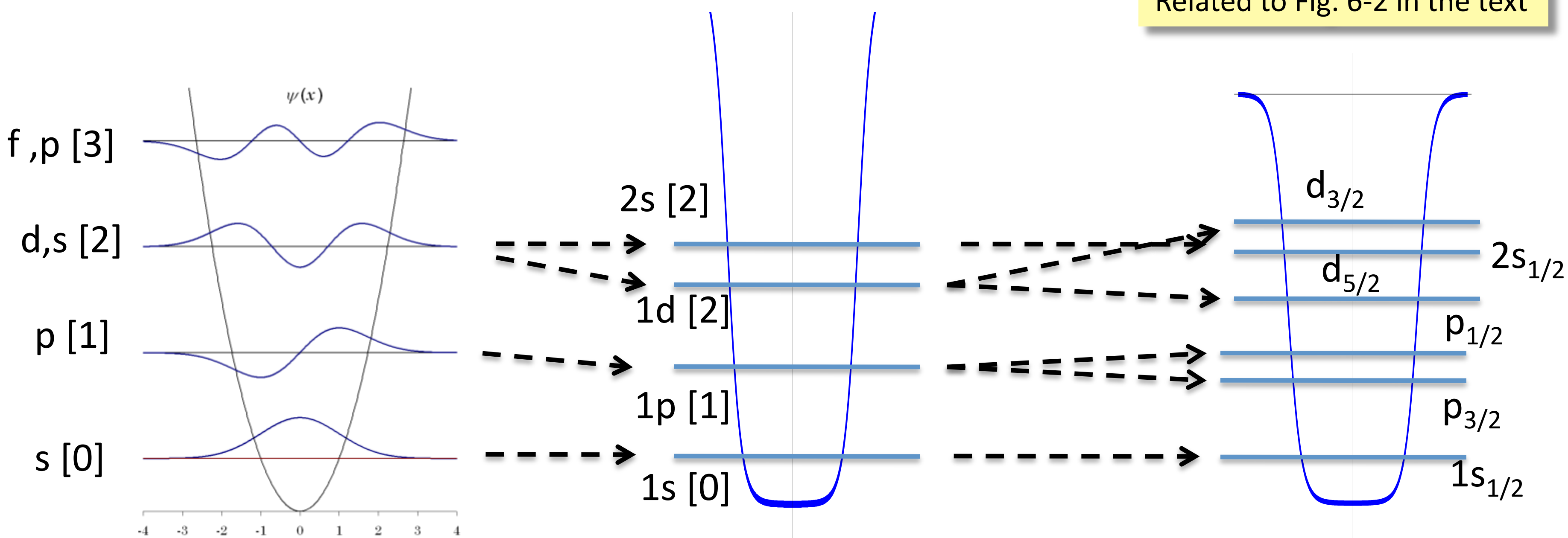
An energy level diagram shows the relative positions and spacing of levels in a quantum mechanical system. The solution for the harmonic potential is well-known with equal spacing between levels.

(1) Flattening out the bottom of the potential energy lowers the energies of states with higher angular momentum.

(2) Making the potential "finite" rather than "infinite" favors states by spin-aligned with orbital motion.

Related to Fig. 6-2 in the text



harmonic potential
equal spacing

flat bottom
lowers by orbital ang. momentum
l = 0 no, 1 some, 2 more

finite potential
lowers by total ang. momentum
$j = l + \frac{1}{2}$ vs. $j = l - \frac{1}{2}$

Nuclear Energy Level Diagram

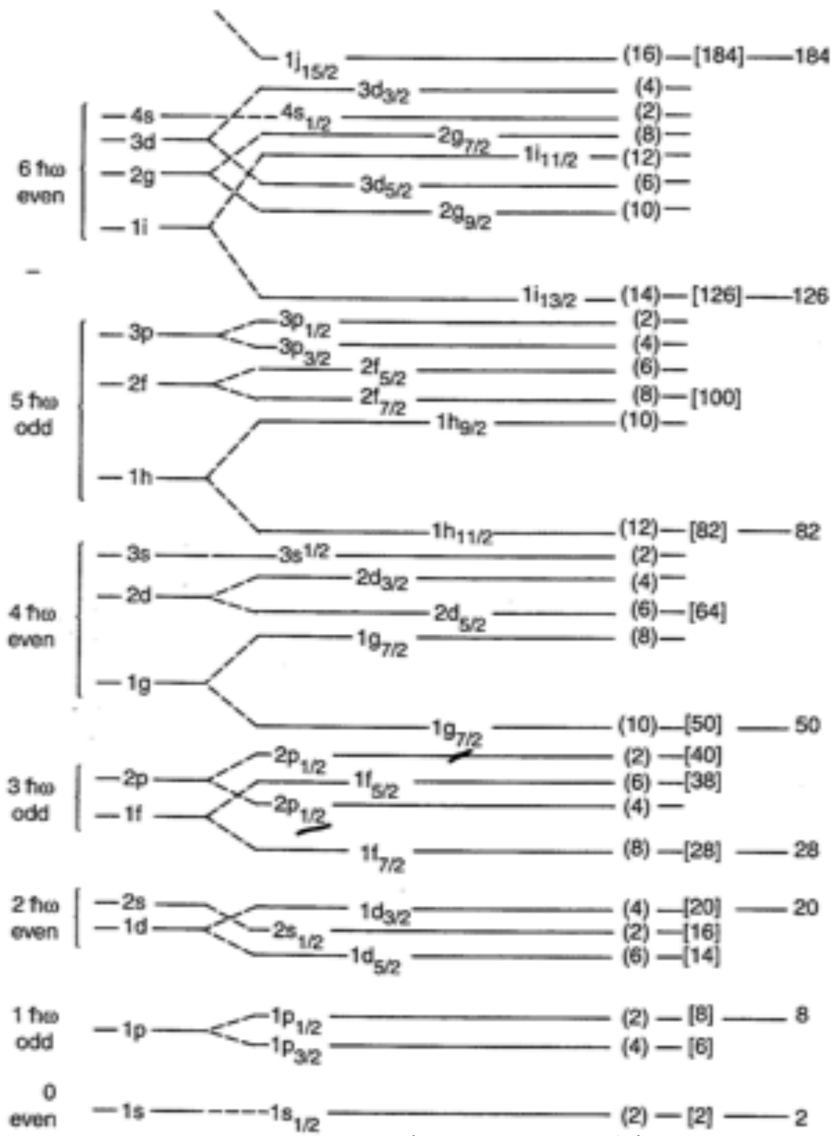


Fig. 6-3 in the text (has two typo's)

The energy level diagram for single particles moving in the nuclear potential for a large nucleus. (the potential energy is different from that for atomic orbitals, thus, we should expect a different energy level pattern.)

1) Orbitals are labeled by orbital angular momentum, *l*, and total spin, *j* (different naming scheme than atomic orbitals)
2) Two neutrons and Two protons go into each orbital (S= +1/2 and -1/2, just like electrons in atomic orbitals)
3) The number of particles in an orbital is 2j+1 (just like electrons in atomic orbitals but you might not have realized this)
4) The total number of particles to fill a nuclear shell are: 2, 8, 20, 28, 50, 82, 126 (different result from atomic shells)

Filling the Nuclear Energy Levels

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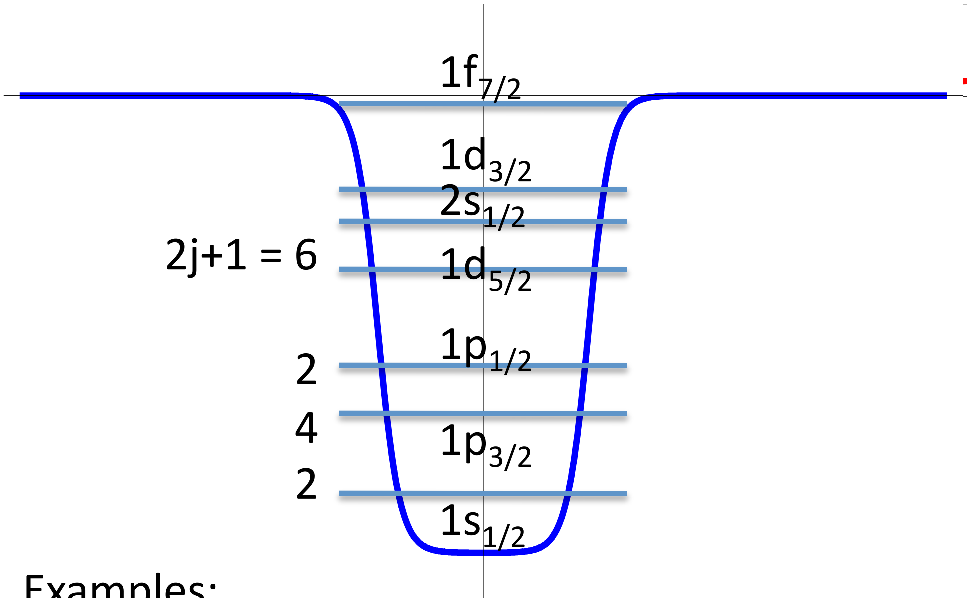
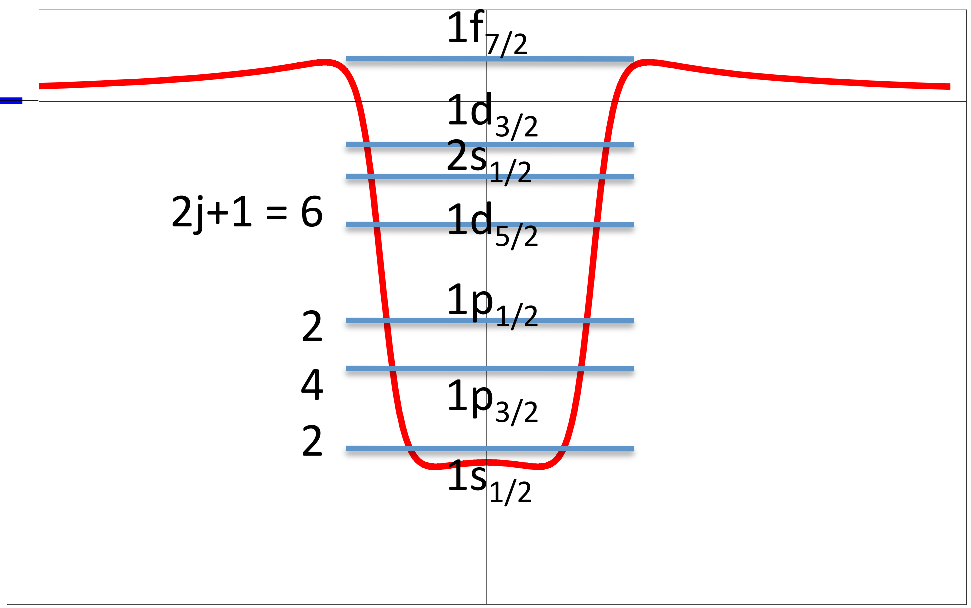
Rules for filling Nuclear energy level diagram:

1) Fill from the bottom
2) (2j) +1 particles per orbital





Examples: