Week 2 Lecture 2 – Radioactive Decay

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Nuclear Decay

-- Decay Law
-- Simplest form of kinetics
-- Sequential Decay (three groups)
-- Radioactive dating, Ages & Natural Activities

Implications of this information (review)

Basics of Nuclear Structure

-- Nuclear sizes & shape
-- Unusual behavior
-- Nuclear potential well
-- Schematic shell model of nuclei

First Homework posted
& due on Monday

Nuclear Decay Rate, Independent Activities

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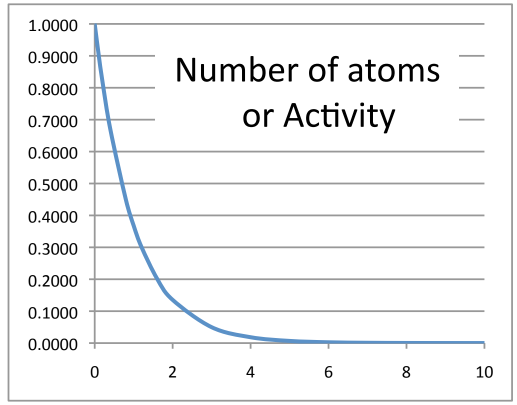
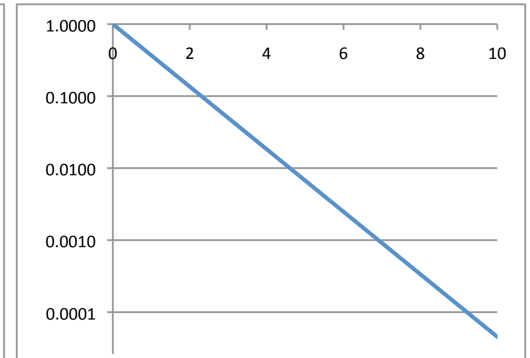
One activity … simple differential equation for the rate of decay

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N \qquad \rightarrow \qquad N = N_0\, e^{-\lambda t}$$

Eq. 3-1 in the text

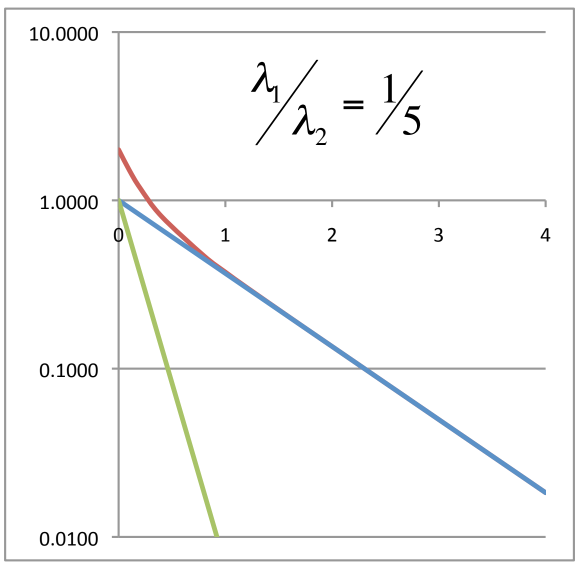


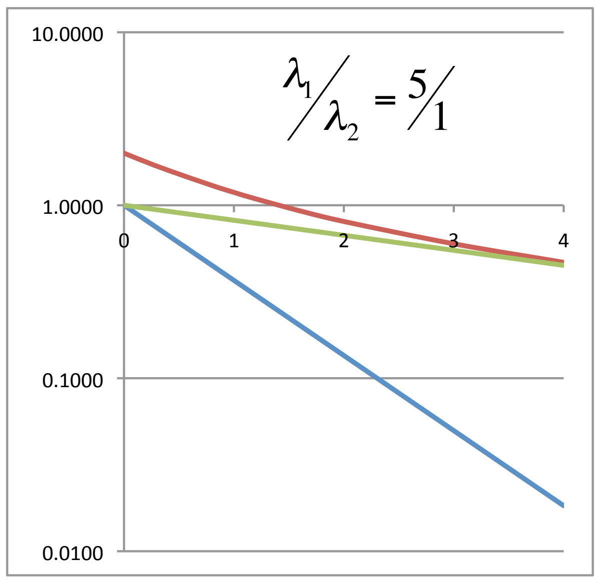


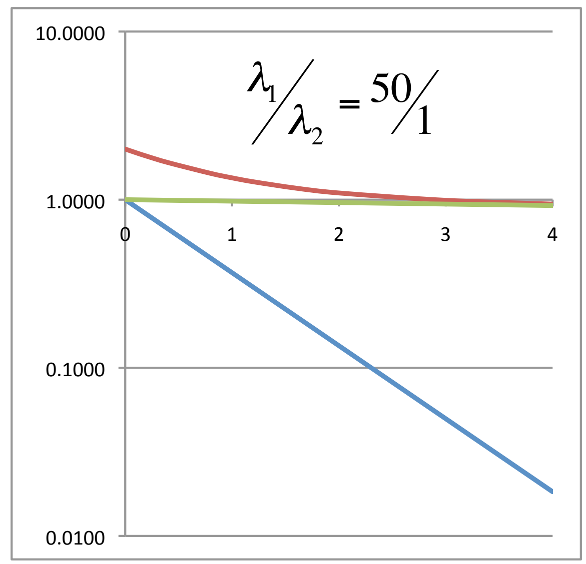
Similar to Fig. 3-1 in the text

Two independent activities …
Simple equations for each decay … the total activity is the sum, of course.

$$A_1 = \lambda_1 N_1 = \lambda_1 N_1^o\, e^{-\lambda_1 t} \qquad A_2 = \lambda_2 N_2 = \lambda_2 N_2^o\, e^{-\lambda_2 t}$$







Similar to Fig. 3-5 in the text

Nuclear Decay Rate, Genetic Relationship

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$$A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B}$$

$$\frac{dN_A}{dt} = -\lambda_A N_A \quad \rightarrow \quad N_A = N_A^o \, e^{-\lambda_A t}$$

Eq. 3-1 in the text

$$\frac{dN_B}{dt} = (\text{Production Rate}) - (\text{Decay Rate})$$

$$\frac{dN_B}{dt} = \frac{-dN_A}{dt} - \lambda_B N_B$$

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B$$

Eq. 3-13 in the text

The parent (A) decays as before without any effect from children…

The children (B) have both a production and a decay rate …

$$N_B(t) = \frac{\lambda_A}{\lambda_B - \lambda_A} N_A^o \left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) + N_B^o e^{-\lambda_B t}$$

Eq. 3-21 in the text

Parent/Child Schematics

$$N_B(t) = \frac{\lambda_A}{\lambda_B - \lambda_A} N_0^A \left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) + N_0^B e^{-\lambda_B t}$$

Eq. 3-21 in the text

$$\lambda_B N_B(t) = \frac{\lambda_B \lambda_A N_0^A}{\lambda_B - \lambda_A} \left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) + \lambda_B N_0^B e^{-\lambda_B t}$$

$$A_B(t) = \frac{\lambda_B}{\lambda_B - \lambda_A} A_0^A \left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) + A_0^B e^{-\lambda_B t}$$

Eq. 3-22 in the text

$$^{99}Mo \xrightarrow{\lambda_A} {}^{99m}Tc \xrightarrow{\lambda_B}$$

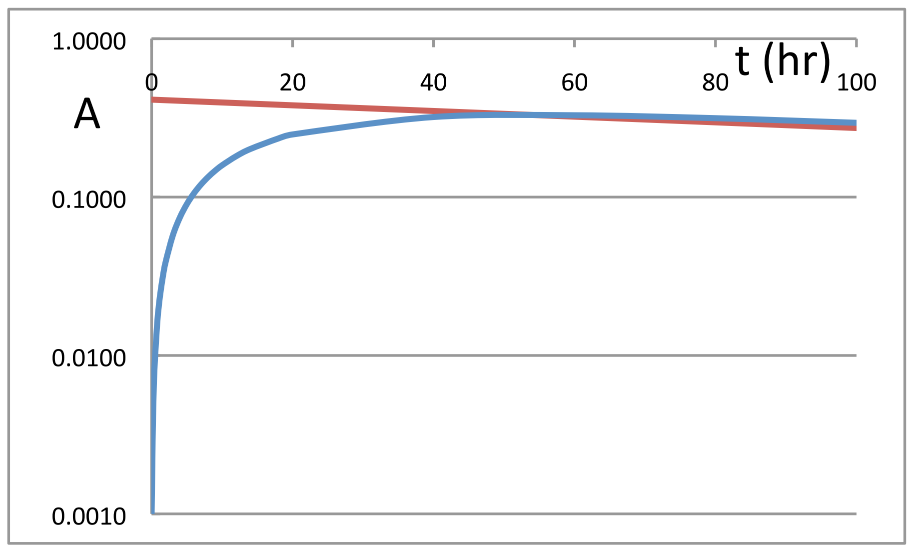
$T_{1/2}$ = 73 hr
N^o_A= 100

6.015 hr
N^o_B= 0

A^o_A = 100*ln 2/73 hr

A^o_B = 0

A^o_A = 0.950 /hr



Parent/Child Schematics

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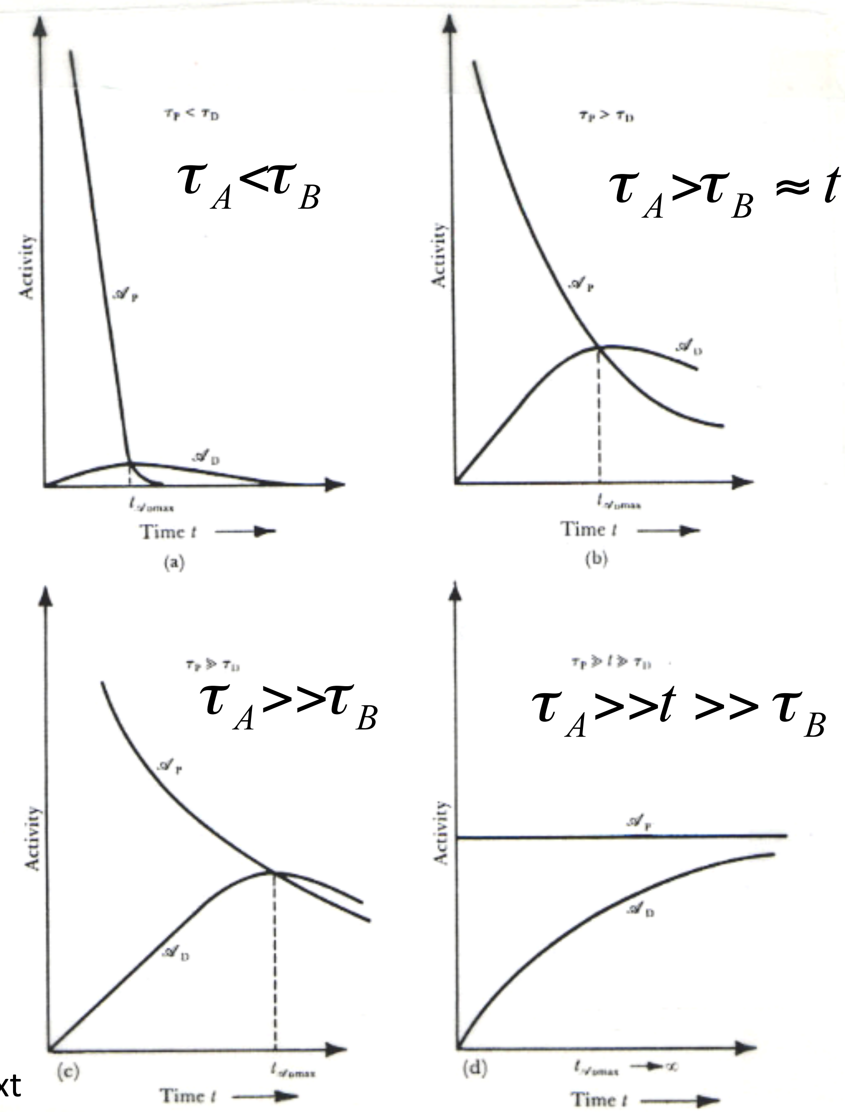

Similar to Fig. 3-8 in the text

Some examples for various cases of τ_A viz. τ_B
N.B. the curves are the activities.

$\tau_A < \tau_B$ … No Equilibrium, most common

$\tau_A > \tau_B$ … Transient Equilibrium, unusual

$\tau_A >> \tau_B$ … Secular Equilibrium, special case

Production in a nuclear reaction

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$$n + A \xrightarrow{\text{ReactionRate}} B \xrightarrow{\lambda_B}$$

$$\frac{dN_B}{dt} = R \quad \rightarrow \qquad N_B(t) = Rt + N_B^o$$

Nuclear reaction with a stable target with a constant production rate (no burn-up of “reactant”).

1) If the product is stable, then a linear increase in product nuclei

$$\frac{dN_B}{dt} = (\text{Reaction Rate}) - (\text{Decay Rate})$$

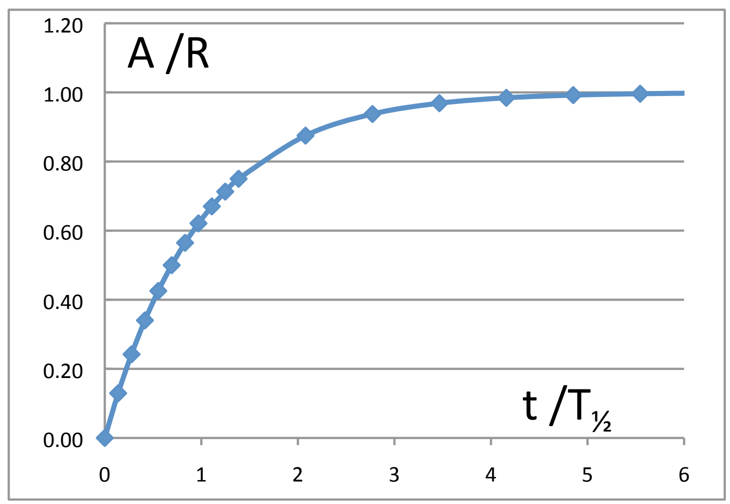
$$\frac{dN_B}{dt} = R - \lambda_B N_B$$

2) If the product is radioactive, then a competition between production and decay.

$$A_B = R\left(1 - e^{-\lambda_B t}\right)$$

For $N^o_B = 0$

For Example:



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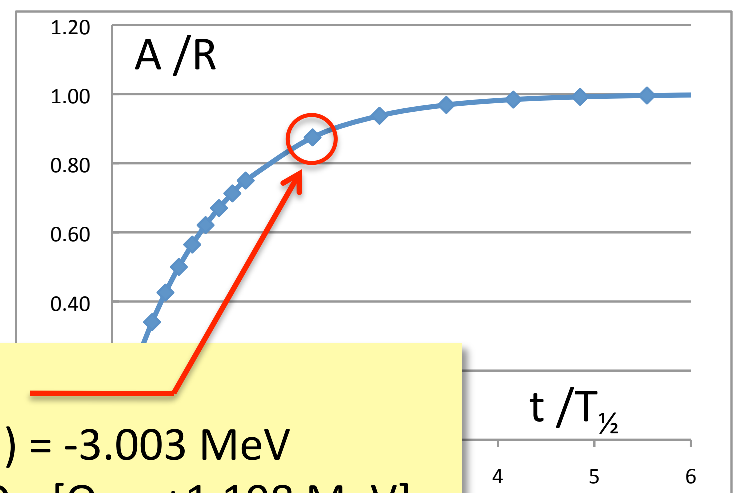
$$A_B = R\left(1 - e^{-\lambda_B t}\right) \qquad \text{For } N^o_B = 0$$

2) If the product is radioactive, then a competition between production and decay.

$${}^{1}_{1}p^{+} + {}^{13}_{6}C \rightarrow {}^{13}_{7}N^{+} + {}^{1}_{0}n$$

$$\lambda_{^{13}N} = \frac{\ln 2}{9.965\,m}$$



How long to run reaction? 3 x $T_{½}$ gives $(1-e^{-3\ln 2})=0.875$

What is Q for this reaction? $\Delta(H) + \Delta(^{13}C) - \Delta(n) - \Delta(^{13}N) = -3.003$ MeV

Write a balanced decay reaction. $^{13}N \rightarrow {}^{13}C^{-} + e^{+} + \nu + Q_{\beta}$ [$Q_{\beta} = +1.198$ MeV]