Week 2 Lecture 1 – Masses & Binding

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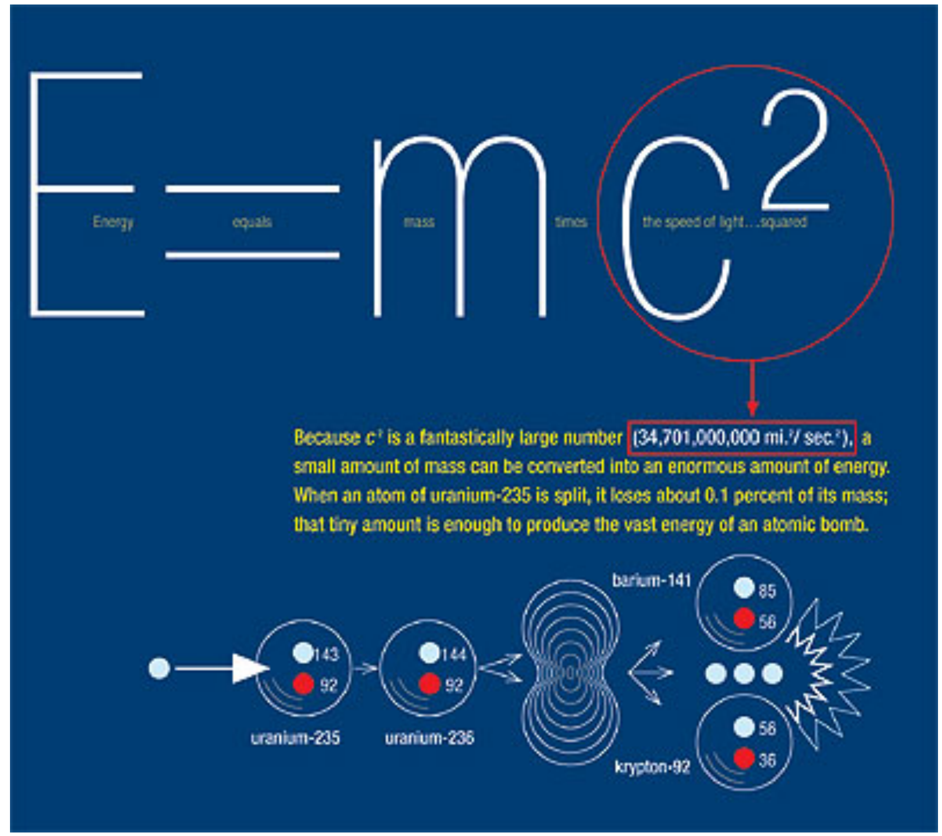
Mass and Energy

-- Einstein's cliché
-- Q values
-- Binding energy viz. Separation Energy
-- "Curve of the Binding Energy"
-- Mass systematics (E-E, E-O, O-E, O-O)
--- Isobaric masses (for beta decay)
--- "Magic numbers" appear
-- Mass model, Liquid Drop
-- Mass surface

Nuclear Decay Equations

-- Decay law
-- examples, Kinetic equations



15 kTons = 63 TJ = 4.2×10^{32} u = 701 kg mass change

Curve of the Binding Energy (stable isotopes)

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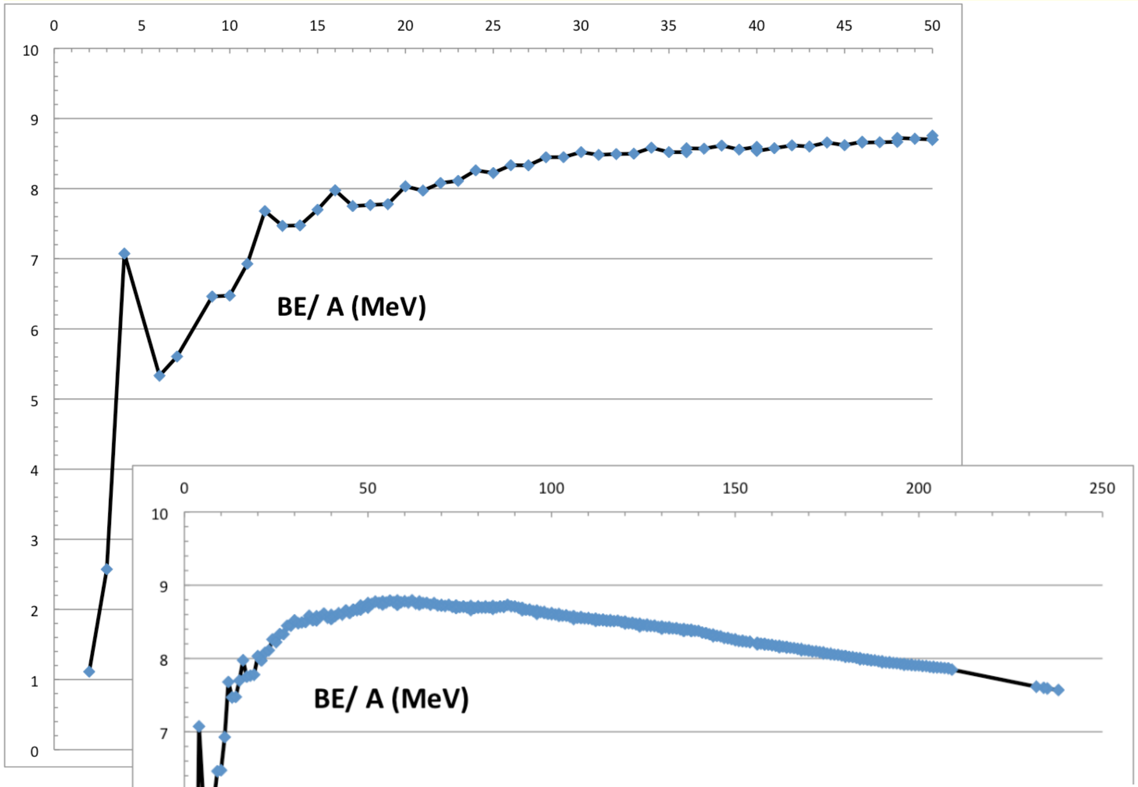

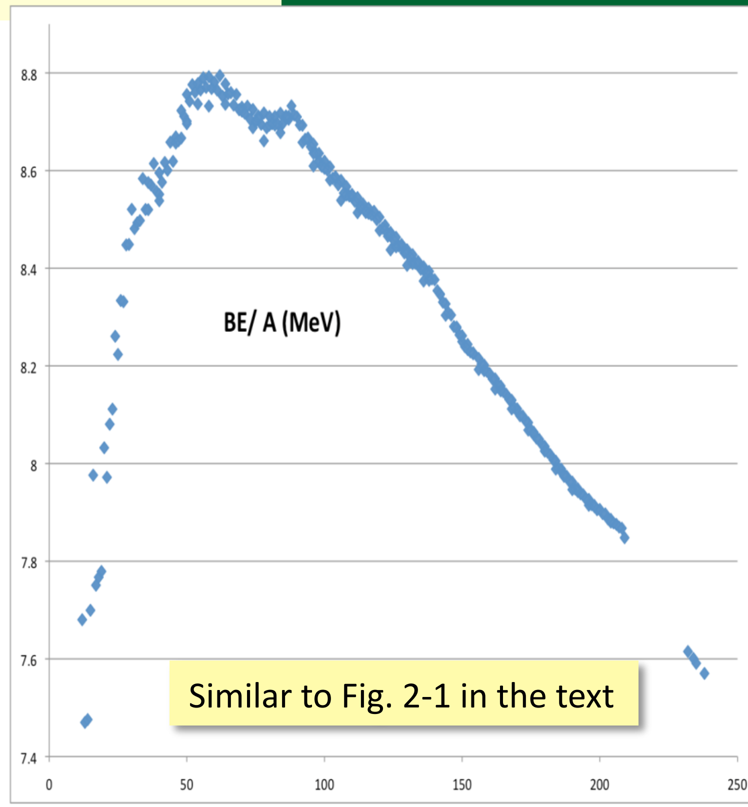

Similar to Fig. 2-1 in the text

Binding energy per nucleon for the stable isotopes:
-- grows rapidly with mass up to ~56, then slowly declines
-- approximately smooth function on large scale
-- definitely not smooth on fine scale
N.B. energy will be released when:
lightest nuclei fuse
heaviest nuclei split

Atomic Masses and Mass Defect (Δ)

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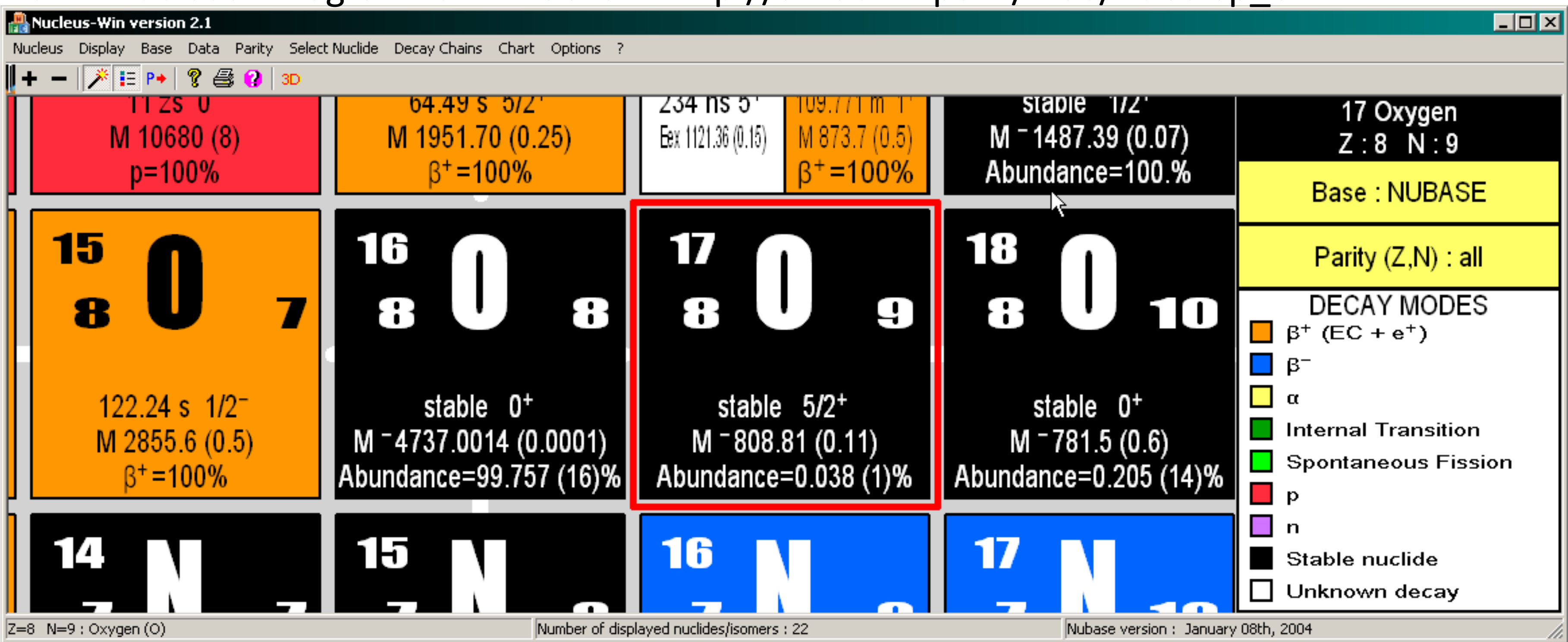
From “Nucleus” Program for windoze: http://amdc.in2p3.fr/web/nubdisp_en.html



Fact: the mass value (u) is always close to the integer mass number.
Reason: the binding energy is small compared to the (Einstein) rest-energy of a nucleon.
(Note that this is also true for electrons.)

Practical use: Define the mass defect (Δ) as the difference from the integer mass number but use energy units (MeV or keV) instead of the traditional mass (u) or the SI unit (kg).

$\Delta = (m - A)\, c^2$ $\quad {}^{18}O$: $(17.999160 - 18)\,u\, c^2 * 931.494\ MeV/u\, c^2 = -0.7825\ MeV$

Systematics of Atomic Masses - 1

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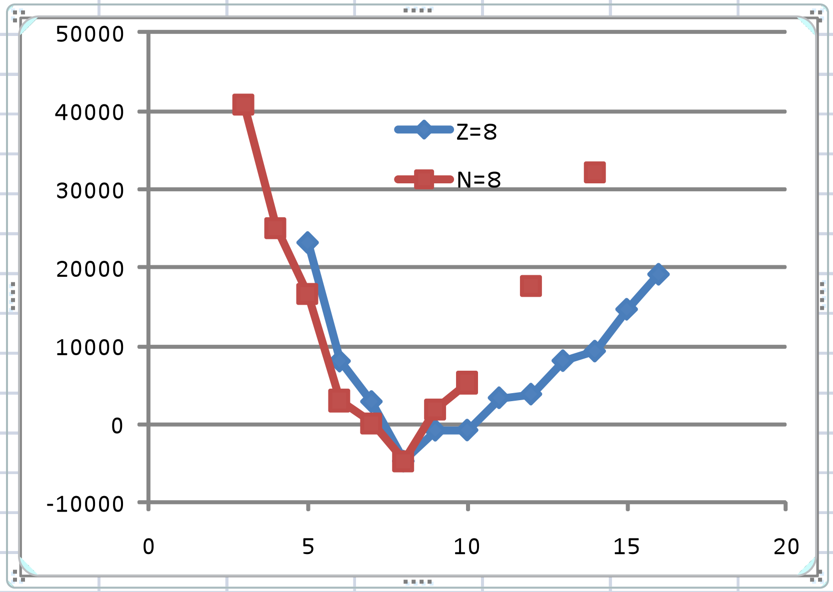
The set of all oxygen isotopes (Z=8): ^{13}O – ^{24}O

The set of all N=8 isotones: ^{11}Li – ^{18}Ne, ^{20}Mg, ^{22}Si

Z=8	Δ (keV)	N=8		Δ (keV)
5	23112	11-Li	3	40797
6	8007.36	12-Be	4	25077
7	2855.6	13-B	5	16562.2
8	-4737	14-C	6	3019.893
9	-808.81	15-N	7	101.438
10	-781.5	16-O	8	-4737
11	3334.9	17-F	9	1951.7
12	3797.5	18-Ne	10	5317.17
13	8063		11	
14	9280	20-Mg	12	17570
15	14610		13	
16	19070	22-Si	14	32160



Data from NUCLEUS-WIN (AME 2004)

Systematics of Atomic Masses – 2a

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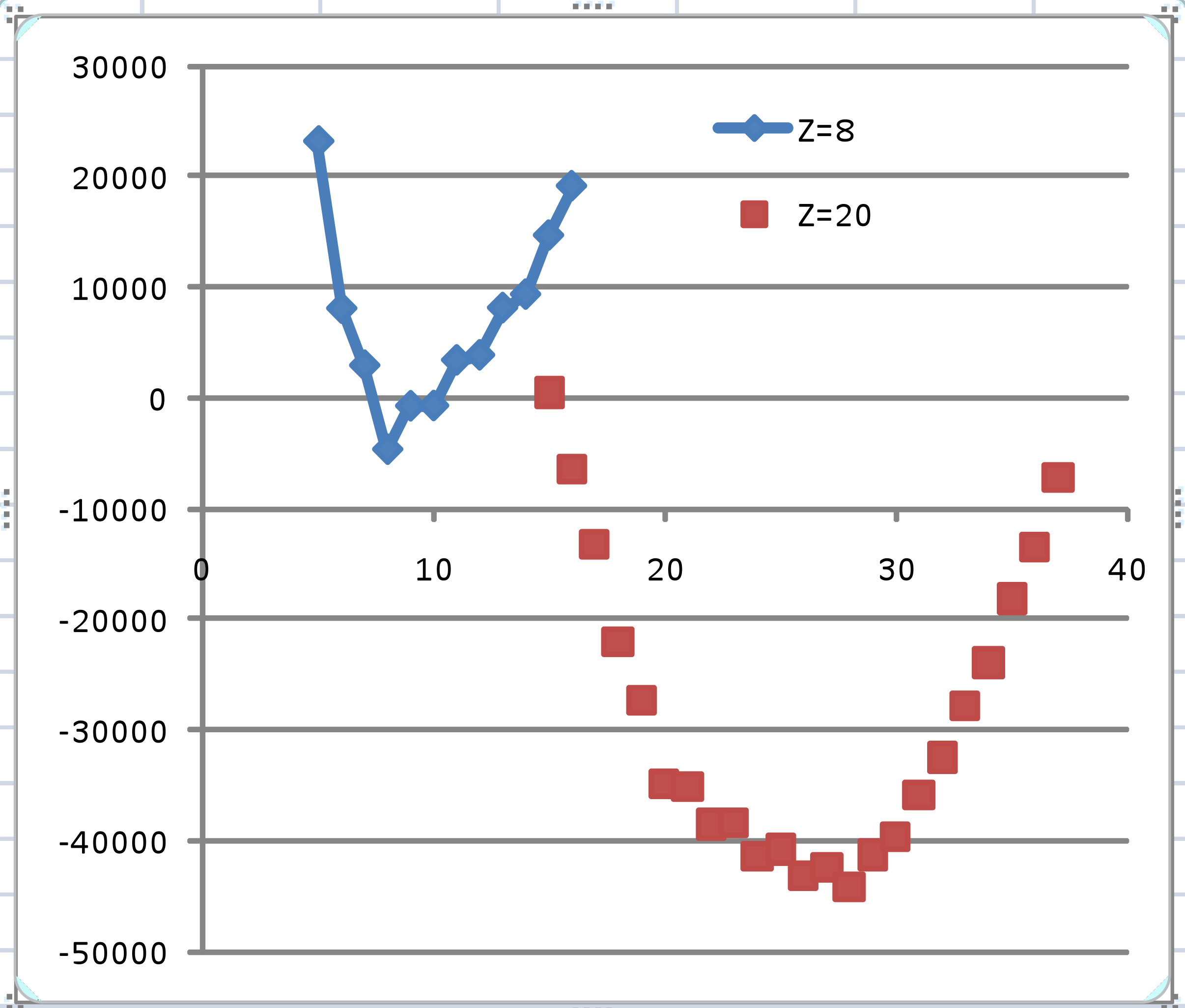
The set of all **calcium isotopes** (Z=20): ^{35}Ca – ^{57}Ca

Z=20	Δ (keV)
15	460
16	-6440
17	-13162
18	-22059
19	-27274.4
20	-34846.3
21	-35137.8
22	-38547.1
23	-38408.6
24	-41468.5
25	-40812
26	-43135.1
27	-42340.1
28	-44214
29	-41289
30	-39571
31	-35860
32	-32510



Systematics of Atomic Masses – 2b

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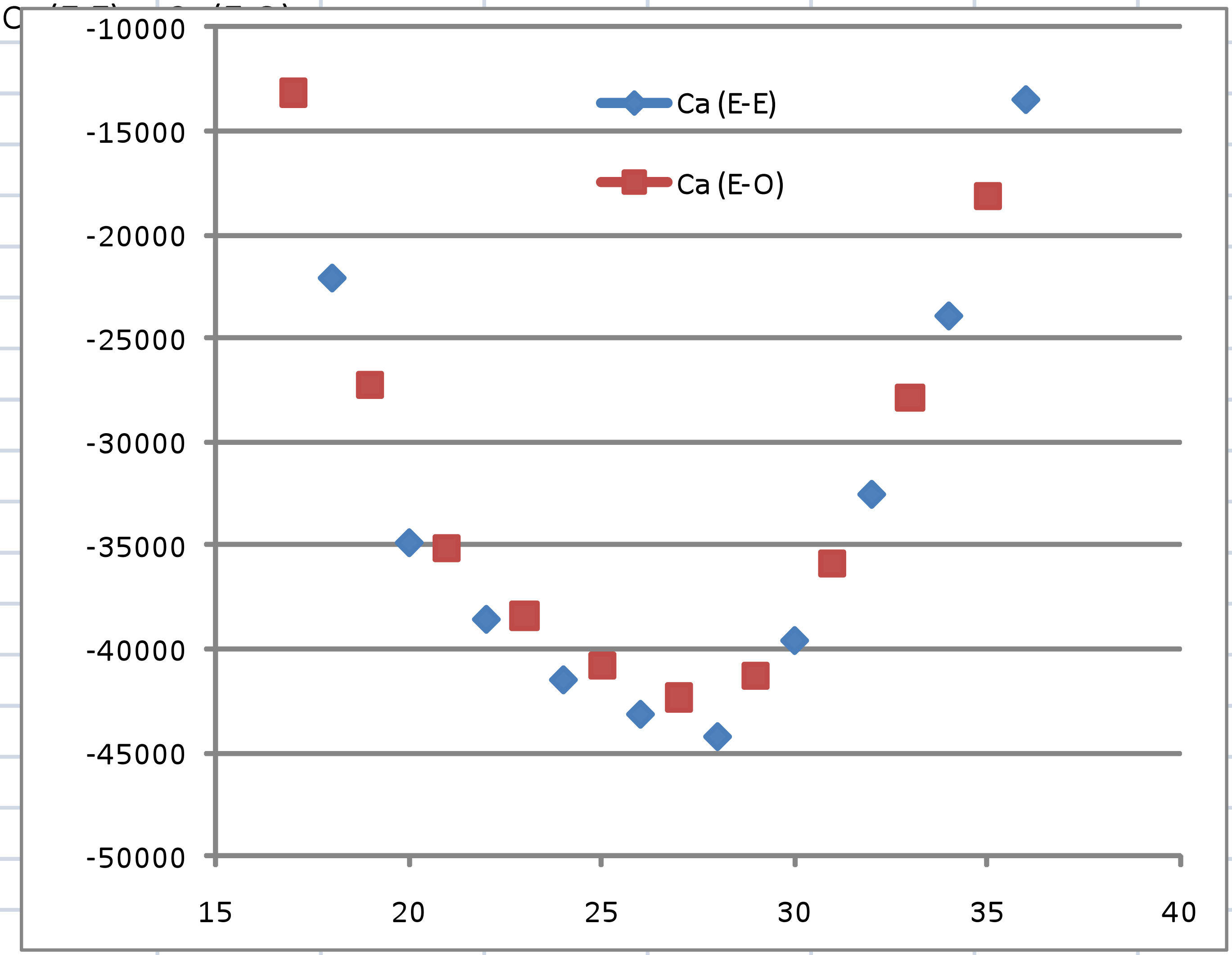
The set of all **calcium isotopes** (Z=20): ^{35}Ca – ^{57}Ca ; Separate by "type"

Z=20	Δ (keV)
15	460
16	-6440
17	-13162
18	-22059
19	-27274.4
20	-34846.3
21	-35137.8
22	-38547.1
23	-38408.6
24	-41468.5
25	-40812
26	-43135.1
27	-42340.1
28	-44214
29	-41289
30	-39571
31	-35860
32	-32510



Systematics of Atomic Masses – 3a

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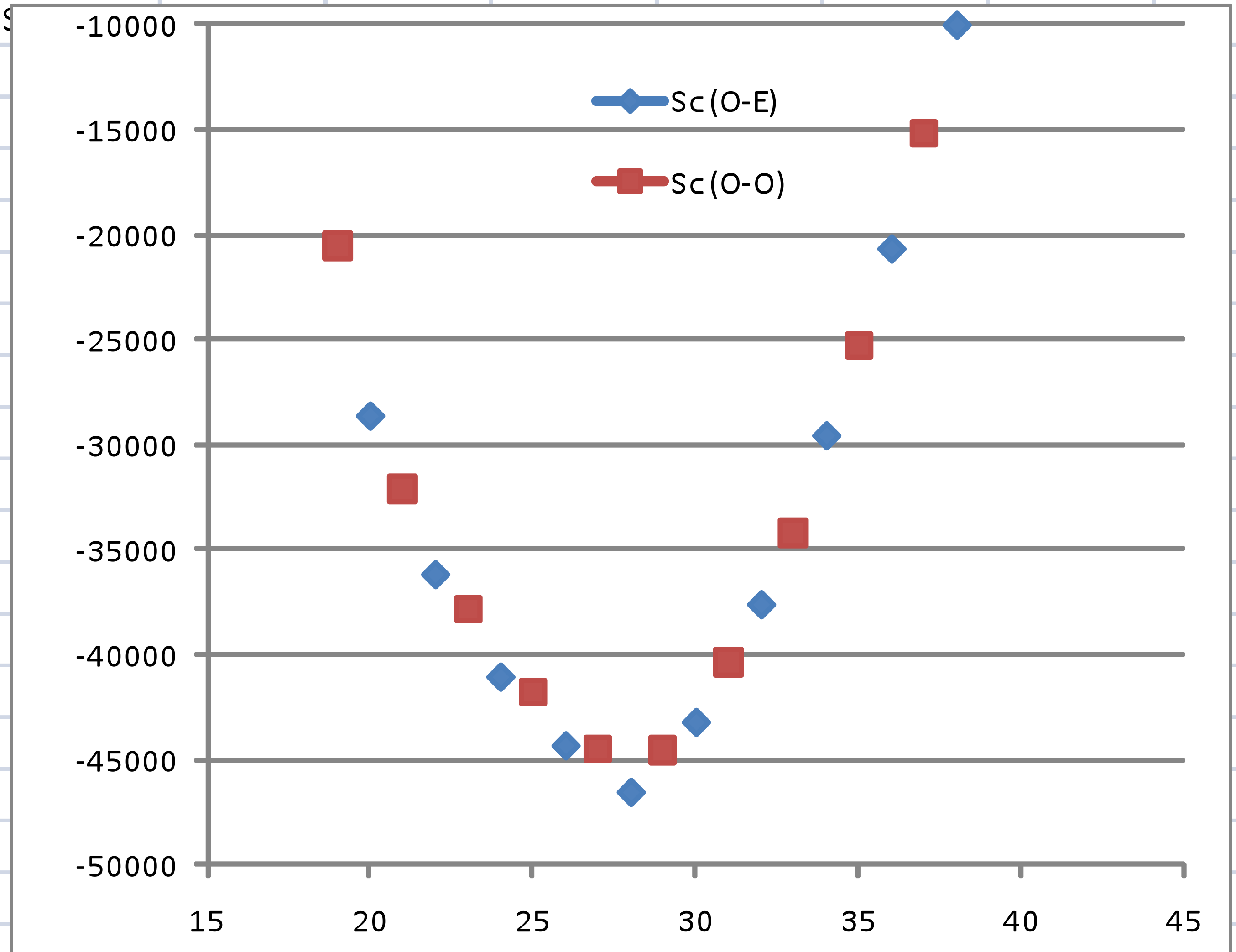
The set of all **scandium isotopes** (Z=21): ^{40}Sc – ^{60}Sc ; Separate by "type"

Z=21	Δ (keV)
19	-20523.2
20	-28642.4
21	-32121.2
22	-36187.9
23	-37816.1
24	-41067.8
25	-41757.1
26	-44332.1
27	-44496
28	-46552
29	-44537
30	-43218
31	-40360
32	-37620
33	-34220
34	-29580
35	-25270
36	-20690



Systematics of Atomic Masses – 3b

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The set of all **scandium isotopes** (Z=21): ^{40}Sc – ^{60}Sc ;   Separate by "type"

Z=21	Δ (keV)
19	-20523.2
20	-28642.4
21	-32121.2
22	-36187.9
23	-37816.1
24	-41067.8
25	-41757.1
26	-44332.1
27	-44496
28	-46552
29	-44537
30	-43218
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34	-29580
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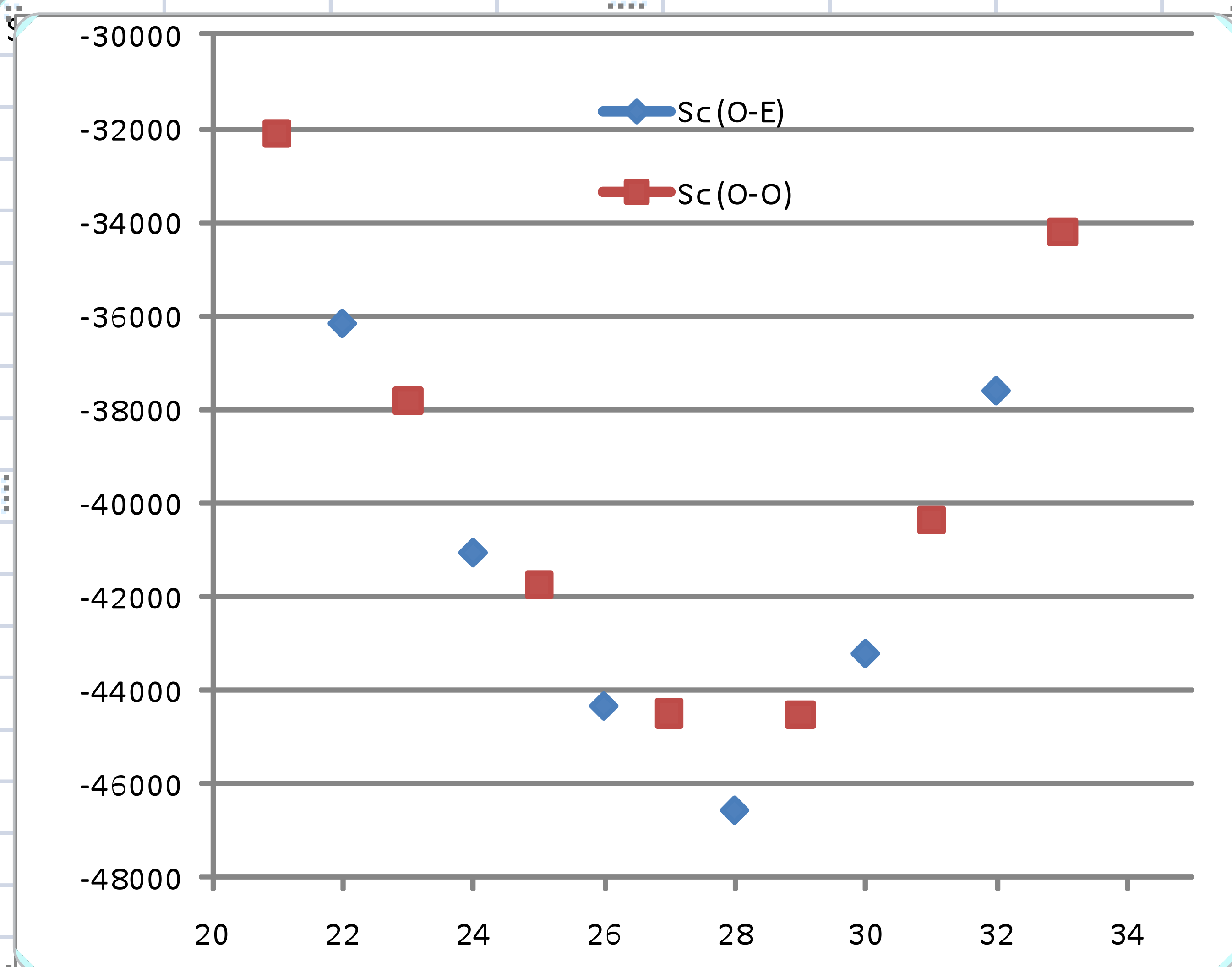

Systematics of Atomic Masses – 4

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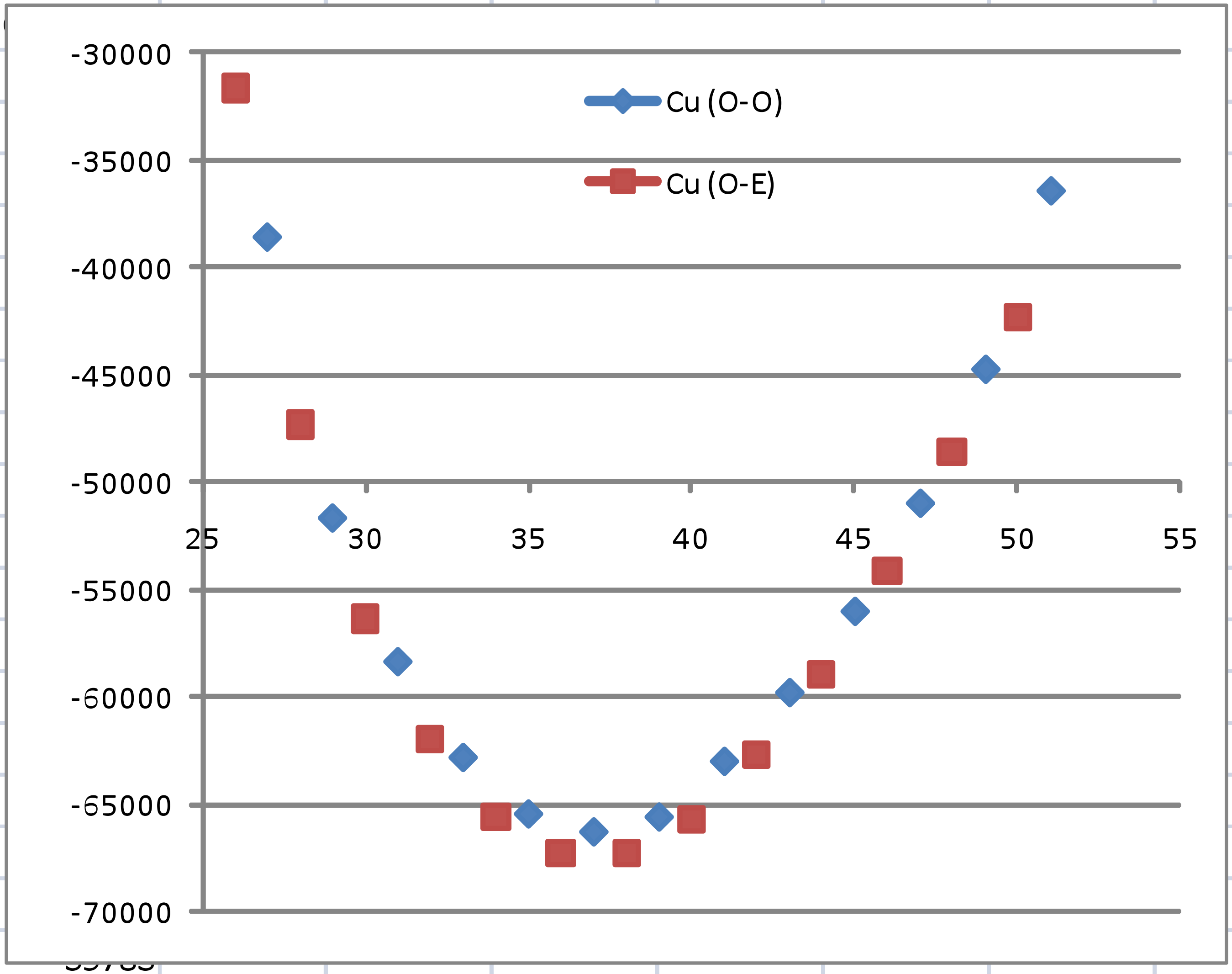
The set of all **copper isotopes** (Z=29): ^{55}Cu – ^{80}Cu ; Separate by "type"

Z=29	Δ (keV)
26	-31620
27	-38600
28	-47310
29	-51662
30	-56357
31	-58344
32	-61983
33	-62798
34	-65579.5
35	-65424.2
36	-67263.7
37	-66258.3
38	-67318.8
39	-65567
40	-65736.2
41	-62976.1
42	-62711.1
43	-59783



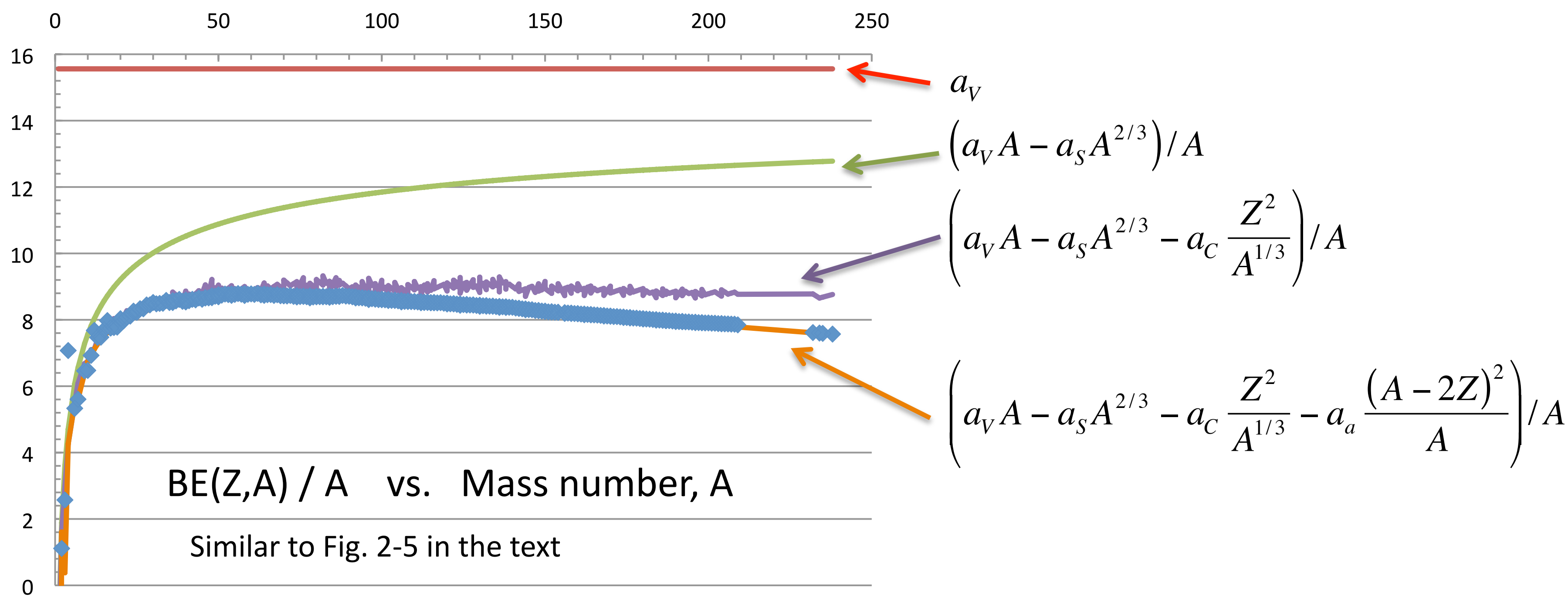
Semi empirical (Liquid Drop) Mass Equation

The total binding energy for a nucleus (and therefore the mass) can be approximated by a constant with corrections:

$$BE(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \delta$$

Sec. 2-7 in the text

Volume term: $a_V A$ binding energy is the same for each nucleon
Surface term: - $a_S A^{2/3}$ binding energy is less for a nucleon on the surface
Coulomb term: - $a_C Z^2 / A^{2/3}$ coulomb force tends to destabilize nucleus
Asymmetry term: - $a_A (A-2Z)^2 / A$ imbalance between neutrons and protons is bad
Pairing term: +/- δ unpaired nucleons destabilize nucleus



Similar to Fig. 2-5 in the text

“Best” value of Z for given A

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$$M(Z,A)c^2 = Zm_H c^2 + (A-Z)m_n c^2 - BE(Z,A)$$

$$M(Z,A)c^2 = Zm_H c^2 + (A-Z)m_n c^2 - \left[a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} \pm \delta\right]$$

Eq. 2-3&4 in the text

$$\Delta(Z,A) = Z\Delta_H + (A-Z)\Delta_n - a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_A \frac{(A-2Z)^2}{A} \mp \delta$$