CEM 483 Fall 2016

Exam 1 Key
October 7

This exam consists of 4 problems on the next six pages. Please examine the booklet to make sure you have a complete examination. A periodic table and sheets containing constants, equations and conversion factors are attached to the back of the exam.

Answer each question in the space provided, continuing on the reverse side of the *same* page if more space is needed.

If a question is not clear, insufficient information is given, or there is an apparent error, please notify a member of the instructional staff immediately.

Pay attention to units and to significant figures of your numerical answers. **Show your reasoning for all problems on the exam!**

1. 20 points	Jin
2. 20 points	Nick
3. 30 points	Jin
4. 30 points	Nick
Total	

Name: ______________

Student #: ___________

Section: ___________

Section 1: Tu 12:40 – 1:30 pm
Section 2: Tu 8:00 – 8:50 am
Section 3: Th 11:30 – 12:20 pm
Section 4: Th 9:10 – 10:00 am

1. Bohr modeled the electronic energy of the hydrogen atom from two fundamental ideas. The first was that the electrostatic force of attraction between the proton and its orbiting electron was balanced by the centrifugal force on the electron arising from its orbital motion.

$$\frac{+e^2}{4\pi\varepsilon_o r^2} = \frac{m_e v^2}{r}$$

The second idea was that the allowed electron orbits needed to be an integer multiple of DeBroglie wavelengths.

$$m_e v r = n\hbar \text{ , where n} = 1, 2, \ldots, \infty \text{ .}$$

a. (15 points) Use these equations to develop an expression for the radius of the electron orbits in terns of the quantum number n and constants.

① $v = \frac{n\hbar}{m_e r}$

into ② $\frac{+e^2}{4\pi\epsilon_0 r^2} = \frac{\cancel{m_e}}{r} \cdot \frac{n^2\hbar^2}{m_e^{\cancel{2}} r^2}$

$\frac{r^3}{r^2} = \frac{4\pi\epsilon_0 \hbar^2 n^2}{m_e e^2}$

$r = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} \cdot n^2$

+10 method
⇒ use both expressions.

+5 ans.

$$\boxed{r = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} \cdot n^2}$$

your reasoning can be done w/ equation(s).

1b. (5 points) What is the uncertainty, or standard deviation, in the momentum of the electron in the second Bohr orbit? Show your reasoning.

$$p = m_e v = \frac{n\hbar}{r} = \frac{2\hbar}{r_2} \qquad r = (\text{constant}) \cdot n^2 \leftarrow \text{we know this exactly.}$$

+4 thought or justification. σ_p or $\Delta p = 0$

$\sigma_p = 0$ +1 ans

2a. (10 points) Is the wave function, $\psi(\theta,\phi) = \sqrt{\frac{3}{8\pi}} \sin\theta e^{-i\phi}$, an eigenfunction of the operator, $\hat{A} = -i\hbar \frac{d}{d\phi}$?

Show your reasoning!

$$\left.\begin{aligned}
\hat{A}\psi(\theta,\phi) &= -i\hbar \frac{d}{d\phi}\left\{\sqrt{\frac{3}{8\pi}} \sin\theta e^{-i\phi}\right\} \\
&= -i\hbar\left\{\sqrt{\frac{3}{8\pi}} \sin\theta \frac{d}{d\phi} e^{-i\phi}\right\} \\
&= -i\hbar \sqrt{\frac{3}{8\pi}} \sin\theta\, e^{-i\phi} \cdot -i \\
&= i^2\hbar\left\{\sqrt{\frac{3}{8\pi}} \sin\theta e^{-i\phi}\right\} \\
&= -\hbar\psi(\theta,\phi)
\end{aligned}\right\} +9.$$

+1

Circle one: yes no

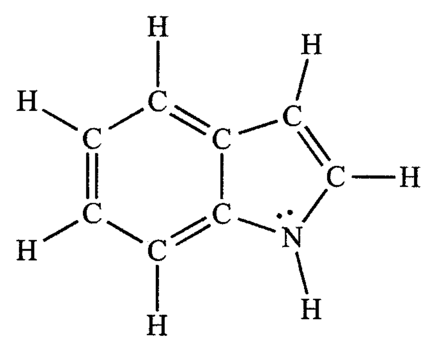
2b. (10 points) The operator, $\hat{A}$, in part "a" represents angular momentum. If a particle is described by the wave function given in part "a", what is the expectation value, or average value of the angular momentum. Assume ψ is normalized over the spatial coord of the problem.

$$\langle A\rangle = \int_\tau \psi^* \hat{A} \psi \, d\tau = -\hbar \int \psi^* \psi \, d\tau = -\hbar$$

+8.

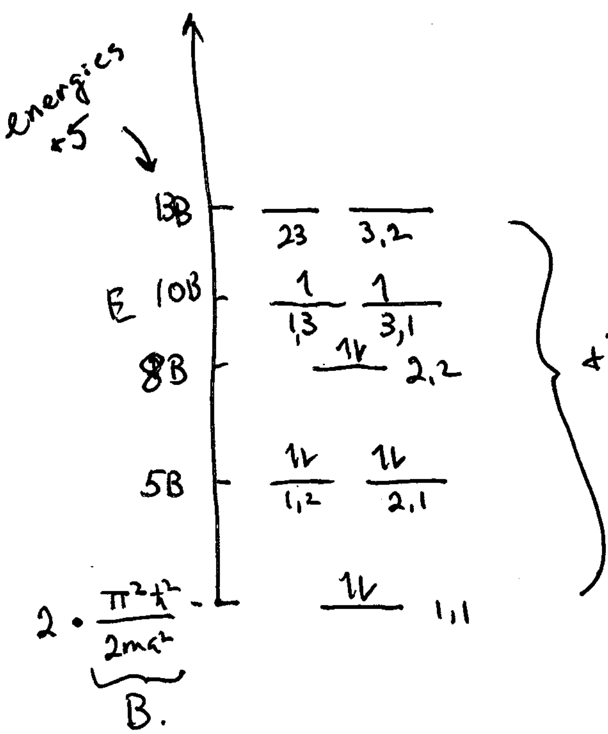
$\boxed{\langle A\rangle = -\hbar \quad +2 \text{ ans.}}$

3a. (15 points) The UV/Vis and fluorescence spectra of proteins are strongly influenced by the indole side chains of the amino acid tryptophan. Modeling the π-electron energies of the indole group as a two dimensional particle in the box, with equal side lengths of 5.0 Å (1 Å = 1 x 10^{-10} m), construct an energy level diagram for the π electrons and populate the energy levels appropriately. The nitrogen atom of the indole group contributes a lone pair of electrons to the pi system, so ~~it~~ the molecule is aromatic. Your diagram should have the energy levels labeled with their quantum number(s) and energy values.



$$E_{n_x,n_y} = \frac{\pi^2\hbar^2}{2ma^2}(n_x^2+n_y^2)$$

+3



+7 labels + degeneracy shown.

3b. (15 points) Using your energy level diagram from part a, calculate the wavelength of light needed to excite the highest energy π electron to the empty energy level that is closest in energy. This is the lowest energy $\pi \longrightarrow \pi^*$ transition and it is observed experimentally at 280 nm.

$$\underbrace{\Delta E = 13B - 10B}_{+5 \text{ E-difference concept.}} = \frac{3\cdot\pi^2\hbar^2}{2ma^2} = \frac{3\pi^2(1.054\times10^{-34}\,J\cdot s)^2}{2(9.11\times10^{-31}\,kg)(5.0\times10^{-10}\,m)^2}$$

$$= 7.16\times10^{-19}\,J = \frac{hc}{\lambda} \leftarrow +5\ E_{rad} = \Delta E.$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\,J\cdot s)(3.00\times10^{8}\,m/s)}{7.16\times10^{-19}\,J} = 2.77\times10^{-7}\,m$$

or

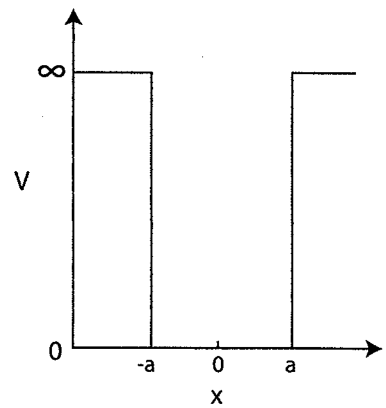
277 nm

+5 value

units? −1

$$\boxed{\lambda = 2.77\times10^{-7}\,m}$$

4a. (15 points) A particle trapped in a 1-dimensional box of length “2a,” centered at x = 0 and with potential energy barriers at x = -a and x = +a, is found in a state described by the wavefunction, $\phi(x) = C(a^2 - x^2)$, where C is a normalization constant. Determine the normalization constant for ϕ(x).



+3

$$\int_{-a}^{+a} \phi^* \phi \, dx = 1$$

$$C^2 \int_{-a}^{+a} (a^2 - x^2)(a^2 - x^2)\, dx = C^2 \int_{-a}^{+a} (a^4 - 2x^2a^2 + x^4)\, dx \leftarrow + \cancel{10}\ 5 \text{ setup}$$

$$= C^2 \left(a^4 x - \frac{2x^3}{3} \cdot a^2 + \frac{x^5}{5} \right) \Big|_{-a}^{+a}$$

$$= C^2 \left(a^5 - \frac{2a^5}{3} + \frac{a^5}{5} - \left(-a^5 + \frac{2a^5}{3} - \frac{a^5}{5} \right) \right)$$

+ 5 mechanics

$$= C^2 (a^5)\left(2 - \frac{4}{3} + \frac{2}{5}\right) = C^2 a^5 \left(\frac{30 - 20 + 6}{15} \right) = \frac{C^2 a^5 16}{15} = 1$$

$$C^2 = \frac{15}{a^5 \cdot 16}$$

$$C = \frac{1}{4}\sqrt{\frac{15}{a^5}}$$

+ 2 ans.

$$\boxed{C = \frac{1}{4}\sqrt{\frac{15}{a^5}}}$$

4b. (15 points) Determine the average kinetic energy for the particle of part "a", remembering that the kinetic energy operator is given by $\widehat{K} = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2}$.

+5 setup

$$\langle E\rangle = \int_{-a}^{+a} \phi^* \hat{K} \phi \, dx = \frac{15}{16a^5}\int_{-a}^{+a} (a^2-x^2) \cdot \frac{-\hbar^2}{2m} \underbrace{\frac{d^2}{dx^2}(a^2-x^2)}_{\underbrace{\frac{d(-2x)}{dx}}_{-2}} \, dx$$

$$= -\frac{15\hbar^2}{32ma^5}\int_{-a}^{+a} (a^2-x^2)(-2)\, dx$$

$$= \frac{15\hbar^2}{16ma^5}\int_{-a}^{+a} (a^2-x^2)\, dx = \frac{15\hbar^2}{16ma^5}\left(a^2x - \frac{x^3}{3}\right)\Bigg|_{-a}^{+a}$$

$$= \frac{15\hbar^2}{16ma^5}\left(a^3 - \frac{a^3}{3} - \left(-a^3 + \frac{a^3}{3}\right)\right) = \frac{\cancel{15}^{\,5}\hbar^2}{{}_4\cancel{16}ma^{\cancel{5}\,2}}\left(\cancel{2a^3} - \frac{\cancel{2a^3}}{\cancel{3}}\right)\cdot\frac{\cancel{4}}{\cancel{3}}$$

$$= \frac{5\hbar^2}{4ma^2}$$

+5 mechanics

$$\boxed{\langle E\rangle = \frac{5\hbar^2}{4ma^2}}$$

+5 ans.