Week 4 Lecture 3 – Predictions from Shell Model

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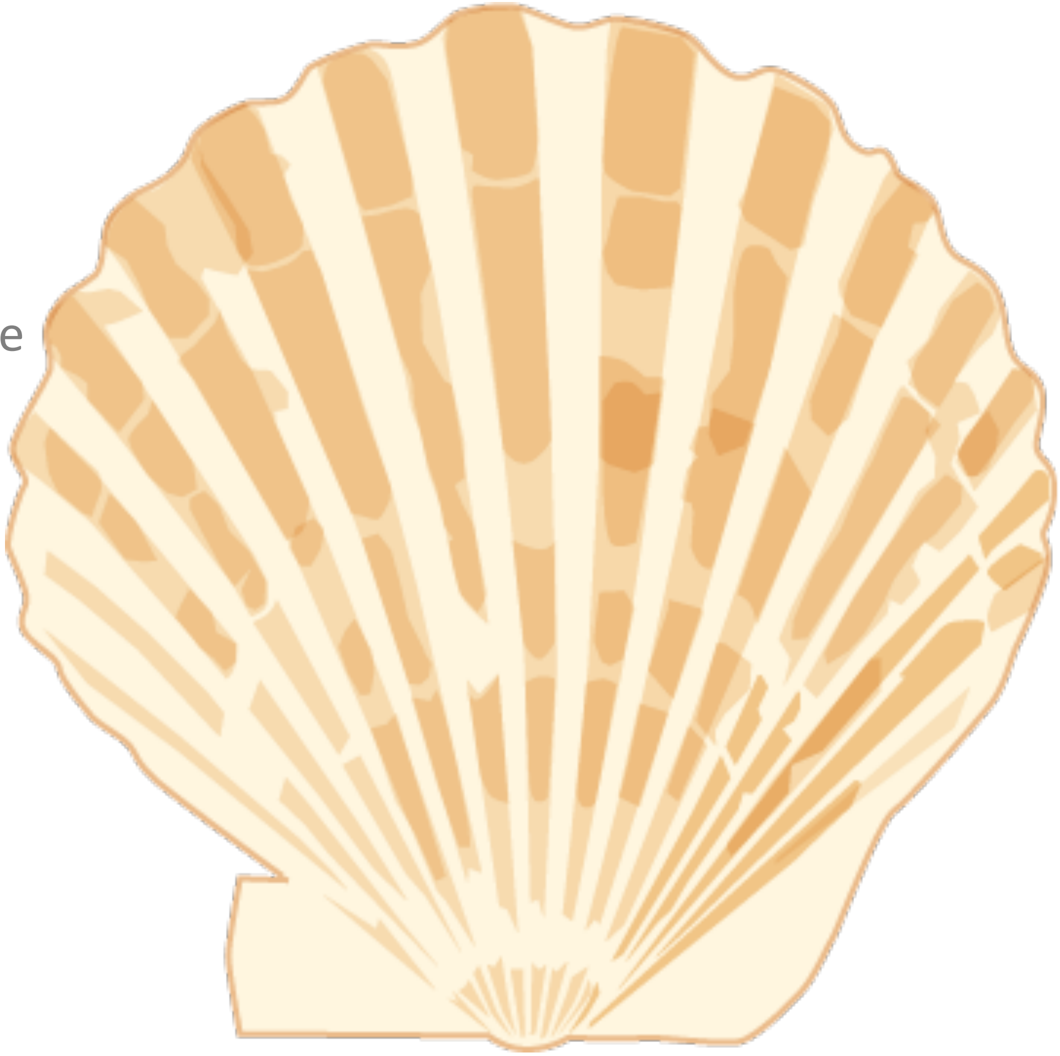
Basics of Nuclear Structure

-- Nuclear sizes & shape
-- Nuclear force viz. Coulomb potential
-- Nuclear potential well
-- Schematic shell model of nuclei
--- Predictions of shell model

Decay Processes

-- Alpha and Beta decay in everyday life
-- Alpha Decay revisited
--- Energetics & Tunneling
-- Beta Decay revisited



3rd Homework due Monday

Repeat: Nuclear Energy Level Diagram

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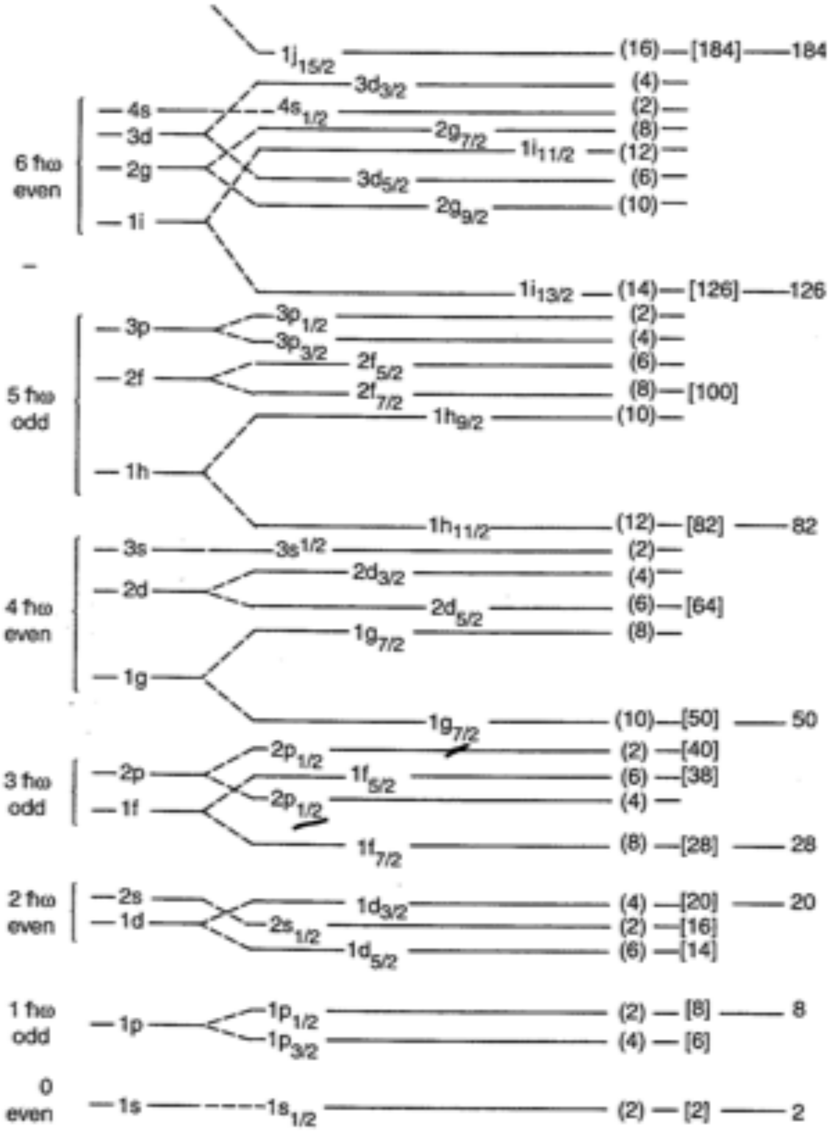
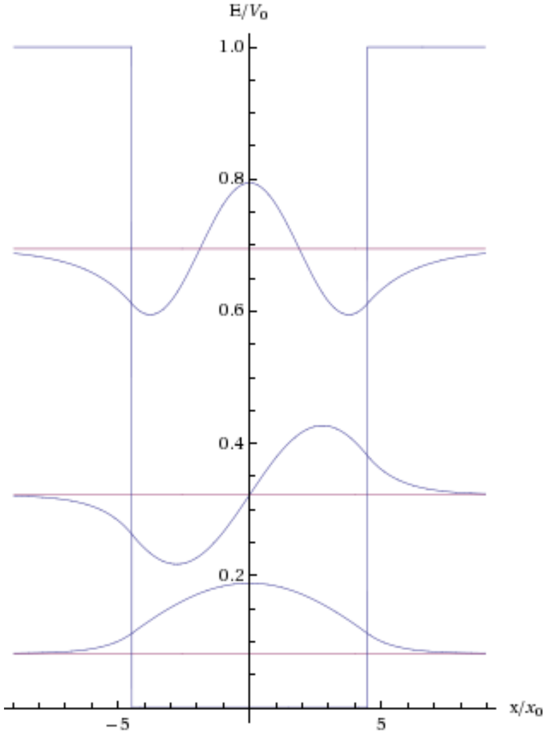


Fig. 6-3 in the text (has two typo's)

The energy level diagram for single particles moving in the nuclear potential for a large nucleus.

Notice that the orbitals are grouped according to their Parity (even or odd). Parity is a fundamental property of a wavefunction that describes their reflection symmetry.

Example:



Wavefunctions in a finite square potential

Ground State Spins and Parities

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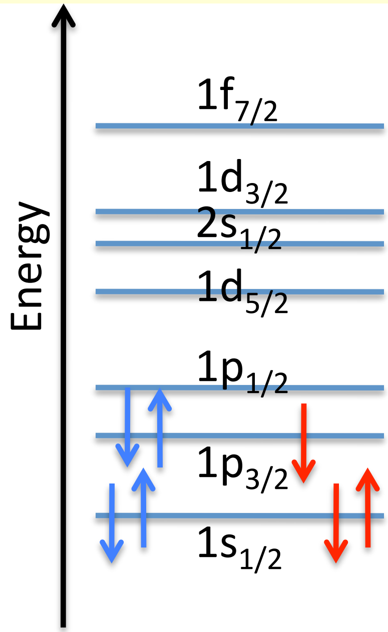


Energy level diagram shows the relative energies of the orbitals available to hold the constituent particles.

The ground state of a (quantum mechanical) system is the lowest energy state of that system and is found by putting all the particles into the lowest possible orbitals in the energy level diagram.

Excited states of a (quantum mechanical) system are all of the states of the system where the particles are NOT in the lowest possible orbitals.

Total nuclear angular momentum, I, is often called "nuclear spin"

The parity of a nuclear state describes the symmetry. It is a product of the parities of the occupied orbitals and is either + or - (even or odd)

Ground State configuration: $${}^{7}_{3}Li_{4} \quad \left[\pi(s^{2}_{1/2}p^{1}_{3/2}) \quad \upsilon(s^{2}_{1/2}p^{2}_{3/2})\right] \leftrightarrow \quad \pi(p^{1}_{3/2})$$

One unpaired particle (a proton) .. Total angular momentum $I = 3/2$, Parity = negative

Lowest energy excited state configuration ? Or what change in configuration will give the smallest change in the energy of this system.

Ground State Spin/Parity of Mirror Nuclei

$${}^{7}_{3}Li_{4} \quad \pi(p^{1}_{3/2}) \quad \leftrightarrow \quad {}^{7}_{4}Be_{3} \quad \nu(p^{1}_{3/2})$$

Notice that the exchange of a neutron for a proton in the mirror nuclei, particularly in light nuclei, will give an odd particle in the same type of orbital. Thus the spin, parity, lowest excited state, etc. should be the same in these two nuclei !

$${}^{17}_{8}O_{9} \quad \left[{}^{16}_{8}O_{8}\right]\nu(d^{1}_{5/2}) \quad \leftrightarrow \quad {}^{17}_{9}F_{8} \quad \left[{}^{16}_{8}O_{8}\right]\pi(d^{1}_{5/2})$$

Q: What is the spin/parity of the lowest excited state in these nuclei?

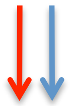
Other Ground State Predictions

All Even-Even nuclei have Spin/Parity = 0^+

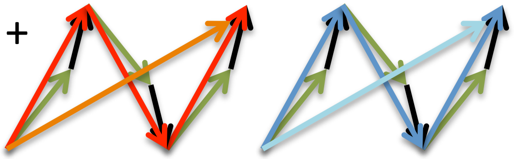
Odd-A nuclei are generally given by the Spin/Parity of the odd-particle.

Odd-Odd nuclei clearly have two couplings of the odd-neutron and odd-proton:
$j_n + j_p$ or $j_n - j_p$ One of these will be the ground state and the other will be an excited state that is very close in energy. For example, three stable odd-odd nuclei:

^{2}H (deuterium): $\pi(s_{\frac{1}{2}}^{1})$ $\nu(s_{\frac{1}{2}}^{1})$ $I = 0$ or 1 nature chooses $I = 1$, Parity = +



^{10}B : $\pi(p_{3/2}^{3})$ $\nu(p_{3/2}^{3})$ $I = 0$ or 3 nature chooses $I = 3$, Parity = +



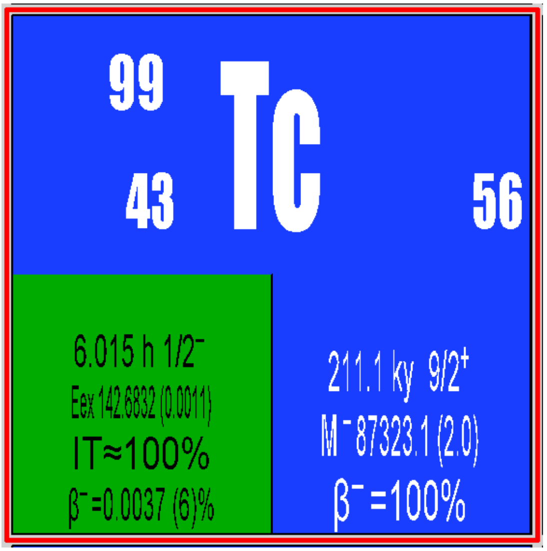
OR $\pi(p_{3/2}^{-1})$ $\nu(p_{3/2}^{-1})$

^{14}N : $\pi(p_{1/2}^{1})$ $\nu(p_{1/2}^{1})$

The ^{99m}Tc example

Revisit a nucleus we looked at before: ^{99m}Tc, recall that the metastable state is an excited state that decays to the ground state by an internal transition (IT). We can use the shell model to predict the spin/parity of the ground state.

Q: What about the “m” state ?



99

43

Tc

56

6.015 h $1/2^-$

Eex 142.6832 (0.0011)

IT≈100%

β^-=0.0037 (6)%

211.1 ky $9/2^+$

M$^-$87323.1 (2.0)

β^-=100%

$$^{99}_{43}Tc_{56} \quad [38]\pi(p^2_{1/2}g^3_{9/2})\,[50]\upsilon(g^6_{7/2}) \;\leftrightarrow\; \pi(g^1_{9/2})\ unpaired$$