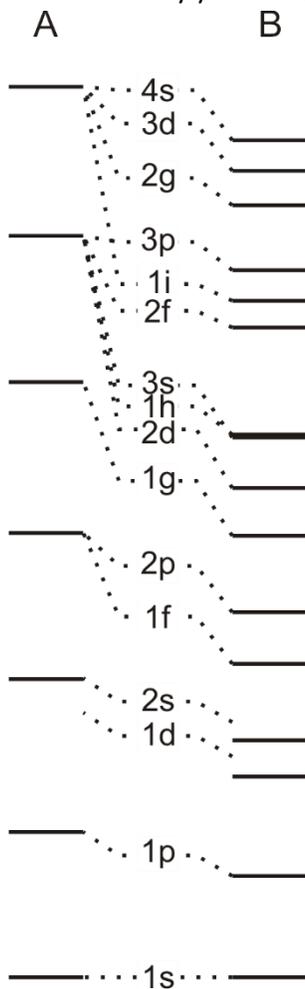


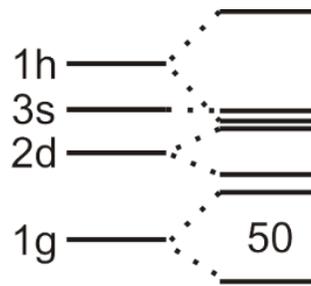
**Nuclear Chemistry**  
**Cumulative Exam**  
**Wed 16<sup>th</sup>, 2013**

This exam is focused on the nuclear shell model. The exam will be graded out of 90 points with the breakdown indicated for each question.

1. (20 points) Consider the three nuclear potentials (1) infinite square well, (2) harmonic oscillator, and (3) Woods-Saxon.
  - a. Sketch the three nuclear potentials in your test book as a function of radius.
  - b. Single particle level diagrams are shown in the figure below using two different nuclear potentials A and B. In your test book label A and B as either the infinite square well or harmonic oscillator potential and describe why you made that choice?



2. (30 points) Reproduce the following figure in your exam book. The right hand side of the figure shows the proton single-particle orbitals in the vicinity of the tin isotopes and includes the effect of the spin-orbit interaction.



- Label all seven levels on the right-hand side of the figure with  $l$  (use the notation s,p,d,f ...) and  $j$ .
- Why does the 3s state not split into two separate levels?
- The expectation of the spin-orbit operator ( $I^*s$ ) can be obtained using the following operator:

$$(I^*s) = 0.5*(j^2 - l^2 - s^2)$$

Based on the level diagram shown above what is the sign of the spin-orbit interaction?

- List the expected spins and parities of the ground state and first three excited states of  $^{133}_{51}\text{Sb}_{82}$ .
  - What ground state spins and parities are possible based on the coupling of an odd-proton with an odd-neutron in  $^{134}_{51}\text{Sb}_{133}$  assuming the odd neutron is placed in the  $f_{7/2}$  orbital?
  - What do you expect for the ground state spin and parity of  $^{134}_{51}\text{Sb}_{83}$ ?
3. (40 points) The following questions (a-h) refer to the Ca isotopic chain and focus strictly on neutrons in the  $f_{7/2}$  single-particle orbit.
- What is the degeneracy of the  $f_{7/2}$  orbital?
  - What are the allowed  $J$  values for two neutrons in the  $f_{7/2}$  orbit? (Hint: Start by determining all allowed combinations of  $m_1$  and  $m_2$ , the projections of  $j_1$  and  $j_2$  on the  $z$ -axis, and then group the results into states with total  $J$ ).
  - What is the ground state spin and parity of  $^{42}_{20}\text{Ca}_{22}$  and why?
  - Using the binding energies provided in the table at the end of this test to estimate the neutron effective single-particle energy of the  $f_{7/2}$  orbital.
  - Based on the answer to part (d) what would you predict for the binding energy of  $^{42}\text{Ca}$ ?
  - Use the discrepancy between the answer for part (e) and the binding energy of  $^{42}\text{Ca}$  in the table to estimate the two body matrix element between two neutrons in the  $f_{7/2}$  single particle orbit coupled to  $J = 0$ .

- g. The first excited  $2^+$ ,  $4^+$ , and  $6^+$  states in  $^{42}\text{Ca}$  are at 1524, 2752, and 3189 keV. Estimate the energies of the two body matrix elements for two neutrons in the  $f_{7/2}$  single particle orbit coupled to  $J=2$ .
- h. What  $J$  values are expected for  $^{43}\text{Ca}$  considering only three neutrons in the  $f_{7/2}$  orbital?

*Table 1: Binding Energy per nucleon and atomic mass for selected Ca isotopes.*

	$^{40}\text{Ca}$	$^{41}\text{Ca}$	$^{42}\text{Ca}$	$^{43}\text{Ca}$	$^{44}\text{Ca}$
Atomic Mass (u)	39.96259	40.962278	41.958618	42.958766	43.955481
Binding Energy/ nucleon (keV)	8551.301	8546.703	8616.559	8600.659	8658.170