1. Show that $\mathbf{L} \cdot \mathbf{p} = 0$ and $\mathbf{L} \cdot \mathbf{r} = 0$.

2. Show that $[L^2, L_z] = 0$. (Hint: Use the commutator relation from problem 3.5).

3. Show that $\langle \hat{L}_x, m_f | \hat{L}_x | \lambda, m_i \rangle = \langle \hat{L}_y, m_f | \hat{L}_y | \lambda, m_i \rangle = 0$. (Hint: One path is to express these operators in terms of the corresponding raising and lowering operators).

4. Consider the states defined by the kets, and represented as column vectors below

\[
\begin{align*}
|1,1\rangle &\equiv (1,0,0)^+ \equiv |+\rangle \\
|\lambda, m_i\rangle &\equiv |1,0\rangle \equiv (0,1,0)^+ \equiv |0\rangle \\
|1,-1\rangle &\equiv (0,0,1)^+ \equiv |\rangle
\end{align*}
\]

The operators $L_x, L_y, L_z, L_+, L_-$, and $L^2$ will then be representable as 3x3 matrices. Construct them.

By explicit matrix multiplication of your matrices verify that $[L_x, L_y] = L_z$.

Hint: comparison with the raising-lowering operator approach to the harmonic oscillator should give you the idea of how to proceed. However, it is crucial to note that e.g. the raising operator on the $|+\rangle$ state annihilates it, since we deal with finite (3x3) matrices.

5. Show that the most probable radius for the electron in the hydrogen atom in the state $|E_n, n-1, m_i\rangle$ of maximum orbital angular momentum is $n^2a_0$.

6. Evaluate the expectation value of $\cos^2(\theta)$ in the eigenstate $|E_n, l, m_i\rangle$ of hydrogen.

(You may just look up the relation for $\cos \theta Y_{\ell m} = C^{+}_{\ell m} Y_{\ell+1,m} + C^{-}_{\ell m} Y_{\ell-1,m}$ stated in, e.g., Arfken or Merzbacher and give me the citation.)

Hint: In doing all the manipulations keep the above symbolic form for the coefficients and only put in their explicit values at the very end. And make use of orthonormality of course.