You may consider this as a homework assignment and may talk to each other about the assignment. Your final answer should be your own work, that is written by yourself, and not copied from others.

We consider a weakly coupled homonuclear spin system. The Hamiltonian can be expressed as in Eq. (VI.C.6) in the notes. The thermal equilibrium density matrix is given by Eq. (VI.C.5). As with the COSY sequence worked out in class, you need only consider the propagation of the $I_{z1}$ term of $\rho(0)$. (The propagation of the $I_{z2}$ term will be the same as that of $I_{z1}$ term with the 1 and 2 subscripts interchanged.)

First consider the pulse sequence $(90)_x$-$t$ where the FID is detected during $t$.

(a) Write the equation for the density matrix $\rho(t)$ analogous to Eq. (VI.D.1).
(b) Evaluate the density matrix right after the $(90)_x$ pulse.
(c) Evaluate the evolution of the density matrix during $t$ due to the Zeeman and chemical shift terms.
(d) Evaluate the evolution of the density matrix during $t$ due to the J-coupling term.
(e) Simplify your expression from (d) by eliminating terms which do not contribute to the observed FID signal.
(f) The operator for the contribution to the FID from nucleus 1 is $M_{x1} + iM_{y1} = n\gamma(I_{x1} + iI_{y1})$
where $n$ is the total number of nuclei of type 1 in the sample. You can see from Eq. (VI.B.2) that $\text{Tr}(I_{x1}^2) = \text{Tr}(I_{y1}^2) = \frac{1}{2}$ and that $\text{Tr}(I_{x1}I_{y1}) = \text{Tr}(I_{y1}I_{x1}) = 0$. Use these expressions and Eq. (VI.B.6) to calculate the contribution to the observable FID from nucleus 1. What are the NMR frequencies associated with nucleus 1?

Now consider the sequence $(90)_x$-$\tau$-$(180)_y$-$\tau$-$t$ where $\tau$ is a fixed time and the FID is detected during $t$.

(g) Write the equation for the density matrix $\rho(\tau,t)$ analogous to Eq. (VI.D.1).
(h) Use your results from (b)-(d) or if you like, recalculation, to evaluate the density matrix right before the $(180)_y$ pulse.
(i) Evaluate the density matrix right after the $(180)_y$ pulse.
(j) Evaluate the evolution of the density matrix at the end of the second $\tau$ period due to the Zeeman and chemical shift terms. After simplification, your expression should only contain two terms. Describe in one sentence what you have derived.
(k) Evaluate the evolution of the density matrix at the end of the second $\tau$ period due to the J-coupling term.
(l) Evaluate the evolution of the density matrix during $t$ due to the Zeeman and chemical shift terms.
(m) Evaluate the evolution of the density matrix during $t$ due to the J-coupling term.
(n) Simplify your expression from (m) by eliminating terms which do not contribute to the observed FID signal.
(o) At what values of $\tau$ is the density matrix in (n) equal to the density matrix in (e)? At what values of $\tau$ is the density matrix in (n) equal to $-1$ times the density matrix in (e)?