Problem Set 5

1. a. $\mathcal{P}(r) \propto e^{-2Ce^2 \ln r / DbkT} = e^{-2\xi \ln r}$

b. $r\mathcal{P}(r) \propto r e^{-2\xi \ln r}$

$$0 \le r \le r_0 \Rightarrow \int_0^{r_0} r\mathcal{P}(r)\,dr = \int_0^{r_0} r e^{-2\xi \ln r}\,dr$$

$$= \int_0^{r_0} r\left(e^{\ln r}\right)^{-2\xi} dr = \int_0^{r_0} (r)(r^{-2\xi})\,dr = \int_0^{r_0} r^{1-2\xi}\,dr$$

$$\left(\frac{1}{2-2\xi}\right)^{-1} r^{2-2\xi}\Big|_0^{r_0}$$

c. The 0 limit could be problematic. This limit is only finite when $2-2\xi > 0 \Rightarrow 1-\xi > 0 \Rightarrow \xi < 1$

d. If cations become closely associated with DNA, this has the effect of increasing b.

For ξ $b = \dfrac{Ce^2}{D\xi kT}$, $\xi = 1 \Rightarrow$

$$b = \left(9.0 \times 10^9 \frac{J\text{-}m}{C^2}\right)(1.6 \times 10^{-19})^2 C^2 / (78)(1.38 \times 10^{-23} J)(310) = 6.9\,\text{Å}$$

So, ~ 3/4 of the negative charges (5.1 Å) of DNA are neutralized by cations

$$\text{So } \frac{(\text{charge DNA + cations})}{\text{charge (DNA)}} \approx \frac{1}{4}$$

e. $\text{protein} + DNA \rightleftharpoons \text{protein-DNA} + x\,Na^+$ (x: unknown)

$$\log \frac{[\text{protein-DNA}][Na^+]^x}{[\text{protein}][DNA]} = \log K_{eq}$$

$$\log K_{obs} + x \log [Na^+] = \log K_{eq}$$

$$\log K_{obs} = \log K_{eq} - x \log [Na^+]$$

Slope $= x \simeq 10.5$

f. Add cations (salt) ⇒ These cations will compete with protein binding. From Equation above, as $[Na^+] \uparrow$ $K_{obs} \downarrow$