Show all units in each step of every calculation. Always make clear with arrows and carats which quantities are vectors.

1. We will do a quantum mechanical calculation of the transition rate induced by radiation between the $k_1$ and $k_2$ states of the radial atom in the $xy$ plane with radius $a_0$. The calculation will be done in a “rotating frame” in which the electric field of the radiation $\mathbf{ε}_{rad} = \mathbf{ε}_{rad} x$. The transition rate is $|\langle ϕ_{k_2} | -\mathbf{µ} \cdot \mathbf{ε}_{rad} | ϕ_{k_1}\rangle|/\hbar$ where $ϕ_{k_1}$ is the initial eigenstate of the atom, $ϕ_{k_2}$ is final eigenstate of the atom, and $\mathbf{µ}$ is the dipole moment operator. You can understand the plausibility of this expression by considering the time-dependent Schrodinger Equation.

a. (15 points) Derive an expression for $\mathbf{µ}$ as a linear combination of $x$ and $y$ components and in terms of $e$, $a_0$, and $φ$.

b. (15 points) Derive an expression for $-\mathbf{µ} \cdot \mathbf{ε}_{rad}$ in terms of $\mathbf{ε}_{rad}$, $e$, $a_0$, and $φ$. Your expression should have complex exponentials rather than sines and cosines.

c. (20 points) Derive the “selection rules”, i.e. the equation(s) which relate $k_1$ and $k_2$ for transitions with non-zero rates.

d. (10 points) Consider that radiative intensity = radiative power/area and consider a 1 W radiation source focused to a square with 1 mm edge length. The magnitude of the radiative electric field can be calculated using the formula that radiative electric field in V/cm $\approx$ square root of radiative intensity in mW/cm². Calculate the radiative electric field in units of V/cm and to two significant figures.

e. (15 points) Calculate the transition time for an allowed transition in units of s and to two significant figures.

f. (10 points) Is your e result qualitatively reasonable or is it not qualitatively reasonable? Provide a reasoned explanation for your choice.