Decay Processes
-- Alpha Decay revisited
--- Energetics & Tunneling
--- Angular momentum
-- Beta Decay revisited

4th Homework due Monday

Extreme Nuclear Shapes,
Oak Ridge National Lab
Energetics of Alpha Decay

Careful investigation of the energetics of alpha emission from heavy nuclei quickly leads to a conundrum. The typical energy released (Q-value) for these decays is on the order of 5 to 6 MeV – this energy is sufficient to use the alpha particles to induce nuclear reactions on the light elements. For example, the first nuclear reaction ever studied in the lab was \( ^4\text{He}^2+ + ^{14}\text{N} = ^{17}\text{O}^+ + ^1\text{H}^+ \) observed by Rutherford (1919) and the first reaction to make a radioactivity, \( ^4\text{He}^2+ + ^{27}\text{Al} = ^{30}\text{P}^+ + ^1\text{H}^+ \), was carried out by the Joliot-Curies (1934). However, the naturally emitted alpha particles have never (and will never) be able to induce nuclear reactions on heavy elements.

\[
V_{\text{Coul}} = \frac{Z_1 Z_2 e^2}{4 \pi \varepsilon_0 r}
\]
Geiger & Nuttall (1911) observed a correlation between the energy of the emitted alpha particle and the half-life for the natural decay series (E-E alpha emitters). The overall correlation is shown below at the left ... but they reported a straight-line correlation similar to that shown below.

\[ \log(T_{1/2}) \propto \log(E_\alpha) \]

Related to Fig. 7-6 in the text (different abscissa) \( (E_\alpha/\text{MeV})^{-1/2} \)
“Tunneling” in Alpha Decay

The explanation for the observation of alpha decay in heavy nuclei comes from a fundamental and general property of quantum mechanical systems. Not only do we have to be careful to have half-integer multiples of the standing waves, we have to pay careful attention to the shape of the potential energy well, including the edge.

A feature of Quantum Mechanical particles and their wavefunctions is that they are only “completely contained” by an infinite potential energy. If the barrier is not infinite then there is always some probability that the particle will cross the barrier. This process is called QM tunneling. The wavefunction in a finite potential will have three parts, a large part inside, an exponentially decaying part under the barrier, and a small part outside.

The probability that the particle gets out – or the decay constant – depends on the product of two terms: \( \lambda = (\text{rate of trying}) \times (\text{function of barrier thickness}) \)

\[
\lambda = f \ T \\
\ln \left( \frac{2}{T_{1/2}} \right) = \left( \frac{v}{2R} \right) e^{-2G} \quad \quad v = \sqrt{\frac{2E}{m}} \quad G \propto \int_R^b \left( \frac{b}{r} \right)^{1/2} \ dr
\]

Details in Section 7.2 of the textbook.
Comparison to half-life data

The general comparison of the theory to the data shows something is missing, calculated $T_{\frac{1}{2}}$ is always shorter than the measured value by about a factor or 50. (calculated $\lambda$ is too large)

Bad assumption: alpha particle was already there inside the nucleus going back and forth trying to get out!

None the less we can estimate the effect of decay between states with different angular momenta ...

Increasing the angular momentum carried by an alpha particle will decrease the amount of kinetic energy available to cross the coulomb barrier.

"Centripetal potential"
Why emit alpha particles anyway?

Recall that the energy available compared to the coulomb barrier for the decay controls the alpha decay process.

Imagine nuclei emitted from $^{238}\text{U}$

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