Nuclear Decay
-- Decay Law
-- Simplest form of kinetics
-- Sequential Decay (three groups)
-- Radioactive dating, Ages & Natural Activities

Implications of this information (review)

Basics of Nuclear Structure
-- Nuclear sizes & shape
-- Unusual behavior
-- Nuclear potential well
-- Schematic shell model of nuclei

First Homework posted & due on Monday
Nuclear Decay Rate, Independent Activities

One activity ...

\[ \frac{dN}{dt} \propto N \]

\[ \frac{dN}{dt} = -\lambda N \quad \rightarrow \quad N = N_0 e^{-\lambda t} \]

Eq. 3-1 in the text

Two independent activities ...

Simple equations for each decay ... the total activity is the sum, of course.

\[ A_1 = \lambda_1 N_1 = \lambda_1 N_1^0 e^{-\lambda_1 t} \]
\[ A_2 = \lambda_2 N_2 = \lambda_2 N_2^0 e^{-\lambda_2 t} \]

Similar to Fig. 3-5 in the text

\[ \frac{\lambda_1}{\lambda_2} = \frac{1}{5} \]
\[ \frac{\lambda_1}{\lambda_2} = \frac{5}{1} \]
\[ \frac{\lambda_1}{\lambda_2} = \frac{50}{1} \]
Nuclear Decay Rate, Genetic Relationship

\[ A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} \]

\[
\frac{dN_A}{dt} = -\lambda_A N_A \quad \Rightarrow \quad N_A = N_A^0 e^{-\lambda_A t}
\]

Eq. 3-1 in the text

\[
\frac{dN_B}{dt} = (\text{Production Rate}) - (\text{Decay Rate})
\]

\[
\frac{dN_B}{dt} = -\frac{dN_A}{dt} - \lambda_B N_B
\]

\[
\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B
\]

\[
N_B(t) = \frac{\lambda_A}{\lambda_B - \lambda_A} N_A^0 \left( e^{-\lambda_A t} - e^{-\lambda_B t} \right) + N_B^0 e^{-\lambda_B t}
\]

Eq. 3-13 in the text

The parent (A) decays as before without any effect from children...
The children (B) have both a production and a decay rate ...
Parent/Child Schematics

\[ N_B(t) = \frac{\lambda_A}{\lambda_B - \lambda_A} N_0^A \left( e^{-\lambda_A t} - e^{-\lambda_B t} \right) + N_0^B e^{-\lambda_B t} \]

Eq. 3-21 in the text

\[ \lambda_B N_B(t) = \frac{\lambda_B \lambda_A N_0^A}{\lambda_B - \lambda_A} \left( e^{-\lambda_A t} - e^{-\lambda_B t} \right) + \lambda_B N_0^B e^{-\lambda_B t} \]

\[ A_B(t) = \frac{\lambda_B}{\lambda_B - \lambda_A} A_0^A \left( e^{-\lambda_A t} - e^{-\lambda_B t} \right) + A_0^B e^{-\lambda_B t} \]

Eq. 3-22 in the text

\[ ^{99}\text{Mo} \xrightarrow{\lambda_A} ^{99m}\text{Tc} \xrightarrow{\lambda_B} \]

- \( T_{1/2} = 73 \text{ hr} \)
- \( N_0^A = 100 \)
- \( N_0^B = 0 \)
- \( A_0^A = 100 \times \ln 2 / 73 \text{ hr} \)
- \( A_0^B = 0 \)
- \( A_0^A = 0.950 / \text{hr} \)
\[ N_B(t) = \frac{\lambda_A}{\lambda_B - \lambda_A} N_0^A (e^{-\lambda_A t} - e^{-\lambda_B t}) + N_0^B e^{-\lambda_B t} \]

Eq. 3-21 in the text

\[ \lambda_B N_B(t) = \frac{\lambda_B \lambda_A N_0^A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) + \lambda_B N_0^B e^{-\lambda_B t} \]

\[ A_B(t) = \frac{\lambda_B}{\lambda_B - \lambda_A} A_0^A (e^{-\lambda_A t} - e^{-\lambda_B t}) + A_0^B e^{-\lambda_B t} \]

Eq. 3-22 in the text

Some examples for various cases of \( \tau_A \) viz. \( \tau_B \)

N.B. the curves are the activities.

- \( \tau_A < \tau_B \) ... No Equilibrium, most common
- \( \tau_A > \tau_B \) ... Transient Equilibrium, unusual
- \( \tau_A >> \tau_B \) ... Secular Equilibrium, special case

Similar to Fig. 3-8 in the text
Production in a nuclear reaction

\[ n + A \xrightarrow{\text{Reaction Rate}} B \xrightarrow{\lambda_B} \]

Nuclear reaction with a stable target with a constant production rate (no burn-up of “reactant”).

1) If the product is stable, then a linear increase in product nuclei

\[ \frac{dN_B}{dt} = R \quad \rightarrow \quad N_B(t) = Rt + N_B^0 \]

2) If the product is radioactive, then a competition between production and decay.

\[ \frac{dN_B}{dt} = (\text{Reaction Rate}) - (\text{Decay Rate}) \]

\[ \frac{dN_B}{dt} = R - \lambda_B N_B \]

\[ A_B = R \left(1 - e^{-\lambda_B t}\right) \quad \text{For } N_B^0 = 0 \]

For Example:
Production in a nuclear reaction

\[
n + A \xrightarrow{\text{Reaction Rate}} B \xrightarrow{\lambda_B} \n\]

\[
\frac{dN_B}{dt} = R \rightarrow N_B(t) = Rt + N^o_B
\]

1) If the product is stable, then a linear increase in product nuclei

\[
\frac{dN_B}{dt} = (\text{Reaction Rate}) - (\text{Decay Rate})
\]

\[
\frac{dN_B}{dt} = R - \lambda_B N_B
\]

\[
A_B = R \left(1 - e^{-\lambda_B t}\right) \quad \text{For } N^o_B = 0
\]

2) If the product is radioactive, then a competition between production and decay.

\[
\begin{array}{c}
\frac{1}{1} p^+ + \frac{13}{6} C \rightarrow \frac{13}{7} N^+ + \frac{1}{0} n \\
\lambda_{13N} = \frac{\ln 2}{9.965 \text{ m}}
\end{array}
\]

How long to run reaction? \(3 \times T_{1/2}\) gives \((1 - e^{-3 \ln 2}) = 0.875\)

What is Q for this reaction? \(\Delta(H) + \Delta(\text{C}) - \Delta(\text{n}) - \Delta(\text{N}) = -3.003 \text{ MeV}\)

Write a balanced decay reaction. \(^{13}\text{N} \rightarrow ^{13}\text{C}^- + e^+ + \nu + Q_\beta\) [\(Q_\beta = +1.198 \text{ MeV}\)]