Nuclear Chemistry
Cumulative Examination Answers

Wednesday, 21 October 2009

Write your answers to the following questions in the order listed. Make sure that the answers are well-organized and self-explanatory. The total number of points on this exam is 50.

1. (2 point each) Give concise and accurate answers to the following questions:
   (a) What is the key feature of two nuclides that are said to be “isotones?”
   **Two isotones have the same number of neutrons.**
   (b) What is the key feature of two nuclides that are said to be “isobars?”
   **Two isobars have the same mass number.**
   (c) What is the relationship between the mean life and half life of a radionuclide?
   **Mean life (\(\tau\)) = Half life / \(\ln(2)\).**
   (d) Write a COMPLETELY balanced equation for the \(\beta^-\) decay of the nuclide \(^{239}\text{Np}\) (Z=93, the first transuranic nuclide to be identified later shown to have a half life of 2.35 days).
   \(^{239}\text{Np} \rightarrow ^{239}\text{Pu}^+ + \beta^- + \nu + Q\)
   Note that after the decay Pu has only 93 electrons from the Np and thus has a positive charge. The \(\beta^-\) is an ordinary electron and the \(\nu\) is an antineutrino.
   (e) The **baryon number** is conserved in nuclear decay. What is a baryon in this context?
   **The total number of neutrons plus protons (i.e., the number of nucleons) is conserved. A nucleon is a baryon.**

2. (5 points each) The \(^{137}\text{Cs}\) nuclide is something of a nuisance because it is strongly produced in the fission of uranium, it has a moderate half life of 30 years and emits penetrating gamma radiation. The ground state intrinsic spin and parity of \(^{137}\text{Cs}\) is \(7/2^+\) and it decays to the stable nucleus \(^{137}\text{Ba}\) that has an intrinsic spin/parity of \(3/2^+\) with a Q-value of 1175.6 keV.
   (a) This beta decay is an example of an allowed Gammow-Teller transition between states. What are the intrinsic spins and their relative alignment of the particles that are emitted in an allowed Gammow-Teller \(\beta^-\) decay?
   **Both the \(\beta^-\) (electron) and antineutron have an intrinsic spin of \(1/2\ \hbar\). In Gammow-Teller decay the spins are aligned to a total spin, \(S=1\ \hbar\).**
   (b) Would you expect this beta decay to go directly from the ground state of the parent to the ground state of the daughter nucleus? Explain why or why not.
   **No, the total spin change from ground state to ground state would be \(2\hbar\). The \(\beta^-\) and \(\nu\) provide \(S=1\ \hbar\) and can not couple \(I_i=7/2\) to \(I_f=3/2\).**
   (c) Suppose that some of the \(\beta^-\) decay goes to an excited state. What is the most likely decay mode of this state and how will the lifetime of this state compare to the \(\beta^-\) decay lifetime?
   **This excited state (most likely \(I=5/2\)) would then decay by gamma ray (or photon) emission. This decay would be much, much faster than the beta decay. Note that the daughter nucleus is stable and beta decay of the excited state is probably excluded.**
A article in the October 15, 2009 edition of the New York Times stated that area surveys of the Hanford nuclear reactor site were being performed to determine the extent of contamination of rabbits (yes rabbits) with $^{137}$Cs. Cesium is a group 1 element and is easily absorbed by biological systems. The surveys were carried out with some detector carried in a helicopter. Describe a plausible radiation detector that could be used to carry out this survey.

The most reasonable detector would be a large volume scintillation detector that is sensitive to the gamma ray emitted from the excited state. For example, one or more large NaI(Tl) crystals could be carried by a even a small helicopter. They do not require any special cooling and are much more efficient than semiconductor detectors.

Make an estimate of the geometrical efficiency for observing a $^{137}$Cs source on the ground with a 15-inch diameter detector carried in the helicopter at an altitude of 500 feet.

$$\epsilon \approx \frac{\pi r^2_{\text{detector}}}{4 \pi r^2_{\text{separation}}}$$

$$\epsilon \approx \frac{(15/2\text{inch})^2}{4 (500\text{ft} \times 12\text{inch/ft})^2} = \frac{(15/24000)^2}{3.9 \times 10^{-7}}$$

3. (5 points each) Large samples of $^{238}$Pu have been used in some spacecraft as heat sources for radioisotope thermoelectric generators colloquially called “nuclear batteries”. The heat created by the decay is converted into electrical power with thermocouples. This isotope decays by alpha emission with a half life of 87.74 years and has a Q-value of 5.593 MeV.

(a) Give a concise reason why the plutonium isotope is much more preferable in this application to $^{60}$Co even though this cobalt isotope is available in much higher quantities, has a shorter 5.27 year half life (higher decay rate), emits two gamma rays, and only a slightly lower total decay Q-value of 2.824 MeV.

The range of alpha particles is very short (<1 mm) in solid metals so it is very easy to absorb all the energy. The range of the gamma rays from cobalt is long (> few cm) in metals and thus a much larger mass would have to be heated by the radiation.

(b) What is the thermal power in Watts emitted by a 1.0 gram sample of $^{238}$Pu?

Power = Joule / sec = Energy/decay * Decays / sec

Energy/decay = 5.593 MeV/decay * $10^6$ eV/MeV * 1.602 x $10^{-19}$ J/eV = 8.96 x $10^{-13}$ J

Decays/sec = $\lambda * \text{Number of atoms}$

Decay/sec = ln(2)/(87.74 yr * 3.14 x $10^7$ sec/yr) * $N_A$/mol * (1 g/$238$g/mol) = 6.37 x $10^{11}$/s

Power = 8.96 x $10^{-13}$ J * 6.37 x $10^{11}$/s = 0.570 J/s = Watts

(c) The $^{238}$Pu is prepared by chemical separation of used (fission) reactor fuel and is contaminated by its famous neighboring isotope $^{239}$Pu that is produced at a higher rate. The $^{239}$Pu builds up over time as a byproduct in nuclear reactors. Write the sequence of three nuclear reactions that take place to produce the problematic $^{239}$Pu isotope. Hint: the production involves the most abundant heavy isotope in the nuclear fuel.

$^{238}$U + n $\rightarrow$ $^{239}$U + Q$_1$

$^{239}$U $\rightarrow$ $\beta^-$ + $\nu$ + $^{239}$Np + Q$_2$

$^{239}$Np $\rightarrow$ $\beta^-$ + $\nu$ + $^{239}$Pu + Q$_3$